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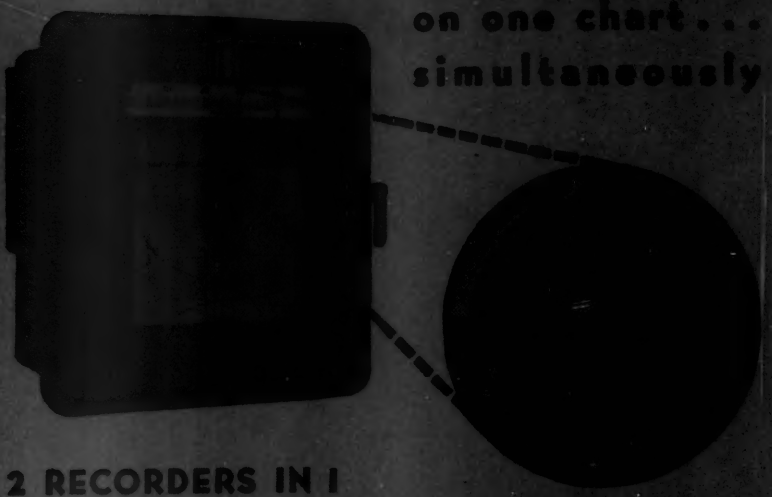

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INDEX TO ADVERTISERS

	PAGE
AG Radio Parts Co.	xi
Ajax Electric Furnace Corp.	ii
Almo Radio Co.	xii
Atlas Photo Engraving Co.	xiv
Beemer Engineering Co.	iv
Bell Telephone Laboratories	Third cover page
Bolger-Parker Co.	xiv
Budd Company, The	Fourth cover page
Buten, M., & Sons	xi
Coleman, W. B., & Co.	xiii
Damon & Foster	xiii
Davidson, P. L.	xiii
De Rewal International Rare Metals Co.	ix
Fehr & Johnson	xiv
Fidelity-Philadelphia Trust Co.	viii
FRANKLIN INSTITUTE, Laboratories	iii
Harris-Dechant Associates	xiii
Herbach & Rademan	xiv
Hess & Barker	xii
Kearney Lumber Co.	xiv
Lancaster Press, Inc.	x
Leeds & Northrup Co.	second cover page
Leopold, Charles S.	xiii
Levy, Max, & Co.	ix
Philadelphia Rust-Proof Co.	ix
Philco	vi
Radio Electric Service Co. of Penna., Inc.	xi
Renninger & Graves	xii
Shanahan & Co.	xiv
Thomson-Porcelite Paint Co.	ix
Yarnall-Waring Co.	x

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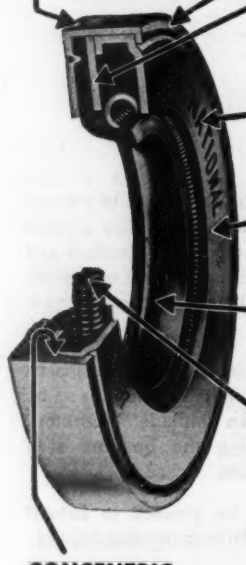

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Vol. 251 — **JUNE, 1951** — **No. 6**

CONTENTS

One Freedom, Indivisible ..... HAROLD E. STASSEN 571
Some Extensions of Elementary Plasticity Theory .. F. EDELMAN AND D. C. DRUCKER 581
Solution of Transients in Active Four-Terminal Networks ..... HERMAN EPSTEIN 607
The Goodness of Fit and the Distribution of Particle Sizes, Part II ..... F. KOTTLER 617
Notes from the National Bureau of Standards 643
The Franklin Institute
- Minutes of Stated Meeting 648
- Committee on Science and the Arts 648
- Membership 649
- Journal of the Franklin Institute 649
- Museum 650
- Library 651

Book Reviews 654
Book Notes 663
Notes from the Biochemical Research Foundation
- Ions and Radicals in Biochemical Processes ..... WILLIAM A. MOSHER 665

Current Topics 606, 642, 664, 675
Index to Volume 251 679

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Journal
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The Franklin Institute
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Vol. 251 JUNE, 1951 No. 6

ONE FREEDOM, INDIVISIBLE [1]

BY

HAROLD E. STASSEN [2]

We live in the most productive society of all history. More than one-third of all of the goods and services of the entire world are produced within the boundaries of our country. This is an amazing fact, all the more amazing when you realize that there live within the boundaries of our forty-eight states only about one-sixteenth of the peoples of the world, and the more amazing, too, when you reflect on the youth of our country in relation to the age of other nations of the world.

It is an amazing fact that should be reflected upon with humility but also with a sense of great responsibility. And I should like to develop with you this evening, as we open our discussion, the thought that this situation is due in no small measure to men like Edward Gowen Budd, in whose honor we meet tonight.

May I say to you that I count it a cherished privilege to deliver the Third Annual Edward Gowen Budd Lecture, and that I am especially appreciative of the opportunity of doing so in the presence of Mrs. Budd, and members of the family and so many of the friends of Mr. Budd, and in these halls, here in this Franklin Institute, with which he had such a long and fruitful and enjoyable association?

I submit that Edward G. Budd's inventive genius and his application of that genius to the concepts of mass production of goods of high quality and reasonable cost and attractive design, which were his life work and which are reflective of the contributions to our economy of a great number of other very important men of his generation, have been keys to the brilliant productivity of our nation. Various aspects of Mr. Budd's personal participation in the amazing development of productivity that America gave the world during the first half of this century

[1] Presented under the Edward G. Budd Lecture Foundation of The Franklin Institute; given in the Lecture Hall at the Stated Meeting held April 6, 1951.

[2] President, University of Pennsylvania, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

have been covered in Mr. G. L. Kelly's first lecture, and various aspects of the continuing resilience of that economy and the opportunities it affords mankind were covered in the second lecture by Mr. William Batt.

I believe it is well for us to recognize that this development has been due to the wise choice by the founding fathers of our country, after one of whom, of course, this Institute was named, of the basis upon which to establish this country—that man should be free and that freedom is indivisible.

By choosing such a basis, they established a way of life in which the opportunity for social, civil, political, religious and economic freedom for the individual man was assured to a greater degree than ever before.

I have little doubt that if that freedom—that indivisible freedom of man—is maintained, the future accomplishments of production in America will be as fruitful and as dramatic and as thrilling as have been the past. I think one of the most significant aspects of the American system has been our emphasis on the fundamental principle that economic freedom is an essential part of the true freedom of man; the principle that man must be free to buy and to sell, to work and to profit, to produce and to build, to invest and to manufacture, to mine and to distribute, to save and to spend, subject only to broad and fair rules of economic growth; the principle that unless that economic freedom is maintained, the other freedoms will not long survive. That is perhaps one of the most essential components of the true, free way of life which the founding fathers of America so clearly desired and so definitely assured us in the governmental and economic and social structure of our country. For they well knew that if a man becomes subservient to the men in authority for what he may eat and what he may wear, for where he may work and what he may earn, for what he may produce and how he shall profit, for what he may build and where he may live, then he cannot truly for long be free in what he may say or write, or how he may worship, or how he may live.

And so upon that basis of economic freedom as a part of the indivisible freedom of man, this amazing economic system, this most productive way of life, this fruitful society, has been developed.

I submit to you that the manner in which the descendants and associates and successors of Edward G. Budd are carrying on is just one small item of proof of the soundness of our confidence in the future productivity and amazing development of our economy—provided the freedom is maintained.

Of course, I would be the last to imply that there will not be some very difficult problems, some hard knocks in the expanding production for defense and for better living in the future in our country. I know full well from direct observation and the reports of friends engaged in

production and in manufacturing and in mining, that the problems of raw materials and of manpower and of design and of governmental relationships are extremely difficult and that they will continue. Nevertheless, I have a deep confidence that these difficulties—of raw materials and manpower and design and regulation, and all that now does occupy production management and business management—will be overcome and that we shall see tremendous productive results which will serve our essential security in armament, and, more important, our progress in the way of life we desire, provided, however, the indivisible freedom of men is maintained.

Therefore, tonight I ask that you turn your attention with me to what I consider to be the three grave threats to our way of life, three dangers which prejudice the continuation of this happy productive economy of ours. In so doing, I turn away from the matters of design and fabrication and production lines. I do turn away from those things, but you will all realize that I do not turn away from the memory of Edward G. Budd, for, as Mr. Kelly pointed out in the first lecture, Edward G. Budd in a high school graduation essay at the age of 17 wrote this significant sentence:

"When we select our occupation, let us be diligent in attending to its duties and ambitions to do well in it, but let us not neglect other duties that pertain to the cultivation of the heart and the mind."

"But let us not neglect other duties that pertain to the cultivation of the heart and the mind." Those who knew him, know that this significant sentence, written in his early years, had a deep and continuing meaning for him. And that is why, in a lecture commemorative of Edward G. Budd it is appropriate that I turn to these three grave threats I mentioned, three areas of tension and uncertainty, which are of major concern to us because of the effect they may have upon our economy and its ability not only to produce material goods but to provide richness and fullness of life for the people of our country and those beyond our shores.

The three areas are these:

1. The interrelationships of men as they are brought closer together by the very accomplishments of modern transportation and mass production and the accompanying growth of great industrial cities.

2. The moral and ethical values of life as these are placed under stress and strain by material accomplishment and by unending modern temptations.

3. The essential requirement for a sound currency as a medium of exchange for the fruits of production, and of other economic interrelationships, and the dangers to that currency presented by national and international financial problems.

Let us discuss briefly together these three, as I see them, threatening situations. You will, of course, immediately have recognized that there are aspects of all three that are very direct consequences of the amazing material and productive accomplishments of our economic system. Take the first, the matter of the interrelationships of men, the way in which they live together. Is it not obvious that men are being brought into ever closer and more continuous and more diverse association, even though they are of very divergent, economic, social, racial and religious background, because of the very nature of this extensive economic system that has been developed in the modern American manner?

I think that that aspect of the problem of inter-racial and inter-religious contacts has not been fully appreciated. We all know that great progress has been made in many aspects of the problem, and sometimes we are inclined to be irritated at what remains to be solved and try to turn away from it, and neglect to think through the consequences of our economic progress and of our scientific progress upon this key question of the interrelationships of men.

To put it another way, I feel very strongly that we have made much more progress in the methods by which men can move through space and come down among peoples of different race and economic status, much more progress in the means of traveling to them or along with them than we have made in developing better understanding between human beings who have different racial or economic or geographic backgrounds.

And so, the first part of our thinking through to a solution of this problem, and therefore our meeting of it in the years ahead without explosion and tragedy, is to recognize the nature of the problem and the consequences of it.

We can start right here at home. Is it not obvious that in this great community of Philadelphia, the people of different parts of the city and of different racial or educational backgrounds have greatly multiplied contacts with each other because they use modern automobiles and transportation and move to different parts of the city to work together in a great modern factory? And is it not also obvious that with our modern means of convenient transportation and many other aspects of our highly developed economic system, the problems are intensified because our people can converge in such great numbers and so easily in the heart of a city to purchase the goods and services that they need?

Clearly the very consequences of our productive genius and our material accomplishments heighten the necessity of thinking through these difficult racial and religious problems and of improving the relationships between men and groups of men with divergent backgrounds. I believe that the effort to think through to a full recognition

of the basic dignity of every human being, regardless of his particular color or status must be made today. We must establish a sound basis for human relationships, the sound basis of mutual respect and an awareness of the necessary divergence of life that comes with association together in a modern community. A superficial approach or mere lip service to this goal of better relationships will be insufficient.

Or transfer the analysis with me, if you will, from that of our immediate community to a situation thousands of miles away, as for example in Asia and in Africa, where I recently had the opportunity of making some personal observations that augmented previous study. I do not know whether you realize that in the vast changes in Asia, the emergence of so many of those countries into independence and sovereignty and self-government raises very different questions from those that confront us at home.

Let me state it this way. In the great arc that extends from Turkey to Japan, through Israel and the Arabian countries, through Pakistan, Afghanistan, India, Burma, Malaya, Indo-China, Indonesia, Philippines, and on up to Japan—the so-called non-communist arc of that part of the world—there live approximately 740 millions of people, or about one-third of the people of the world. Fewer than 2,000,000 of the 740,000,000 are white, and almost every country in which those 2,000,000 live has emerged into sovereignty and independence and faces stupendous problems of self-government.

Therefore, when I speak of this matter of relationships of men toward each other—and I want to dramatize it and emphasize it—I say to you that in that part of the world the question is not how the whites are to treat the coloreds; the question now is how the coloreds are to treat the whites.

Today, in that part of the world, color is one of the most important and vital words. It represents a highly explosive situation filled with plural complexities that have tremendous implications. We will oversimplify the situation if we think merely in terms of those hundreds of millions. But we must realize that their opportunity for progress, for adequately increasing the output of their mills, mines, and fields, depends upon whether there can be worked out an understanding between colored and white a code of sensible relationships by which those who are white can contribute their technical knowledge, their scientific ability, their skill in management. And that is what must be done if the hundreds of millions living in that area are to make sound advances in their standards of living.

In Africa itself, amazing new discoveries of raw materials are made every year. Rich deposits have been found, not only of gold, for which South Africa has been so significant in the past, not only of diamonds, but of uranium, and other metals and materials, and now coal. There again, we have an area in which the relationships of men, their under-

standing of one another, and their mutual adjustment on a sound, dignified basis, will tell whether there is to be world progress in production, in standards of living, and a turning back of the threat of totalitarian domination, or whether, in a chaos of misunderstanding and bigotry, freedom is lost, production is lost, and the progress of the whole world, including America, is set back.

That is why I feel so strongly and so deeply that all of us—you in industry and finance and the other activities of a great city and we in education—have a responsibility to think through these difficult problems and to use all the ingenuity we possess to develop sound methods for improving the relationships of men to other men. That will be one of the keys to the future success of our way of life.

Let me make it clear that I am not one of those who feel that by any legislative acts, or any decrees, or any sudden decisions, you can solve these delicate, difficult, deep-rooted problems. But I am confident that only by the realistic recognition of the problem—on both sides of it and on every side of it—do we have a chance or a hope of creating sound, constructive human relationships that will speed the progress of our society.

Of course, very closely related to all this is the question of what moral and ethical values we shall live by. We have many examples today of low moral standards. Countless people, through the new magic of television, followed the recent Kefauver hearings, and many of them were shocked by what the hearings disclosed concerning present moral and ethical standards. On the one hand, a very interesting exhibition; on the other hand, a very tragic situation.

Then, of course, there are those startling and disappointing disclosures of young men bribed by gamblers to throw basketball games, and the disheartening reports of increasing juvenile delinquency, all of which should give us deep concern for the moral and ethical foundations of our nation.

Such situations clearly do not affect only that percentage who are delinquent, but rather the whole fabric of our society, because if the moral fabric of a society wears thin, then beware lest the society itself be rent asunder.

Specifically relating this problem to a free way of life, I will remind you that in earlier generations it was clearly recognized that a successful free society is possible only if there is present in the individual lives of the overwhelming majority of the society a moral and ethical basis that compels the exercise of self-restraint, of inner control, and does so entirely apart from any laws or regulations or policemen or courts. In other words, if the continuance of a society depends upon the careful definition in law and the enforcement by the police of the code of conduct of the society, then of necessity the specifications must become

so detailed, the enforcement so vigorous and ruthless, that freedom is gone.

I do not think there is a sufficient understanding of the fact that when you destroy the moral foundation of a society as the communists do, you are almost inevitably driven to the kind of ruthless punishment and police state that we know is the manifestation of their way of life. But if you glance back on our great development in this country, you will clearly recognize the fact that from Colonial days to the present the overwhelming majority of our people did not need to know what the law was, but had, rather, within themselves the regard for fellow human beings that gave them a standard of conduct that made it possible for them to live together with a minimum of compulsion and a minimum of legal definition.

When we think of the generations ahead, we must see with equal clarity a very great need for strengthening through education the moral fiber of the nation and of imparting high ethical standards. I am one of those who feel that in this respect we in education have a very strong responsibility. Not that you can look to the school as a substitute for the home or for the churches and other character-building organizations. Nevertheless, it is a fact that one-half of the children of America are now growing up without any contact whatsoever with any church or similar character-building organization, and that most of that half are in homes which themselves have serious deficiencies in their moral standards. Where then are these boys and girls to receive the fair chance to understand moral and ethical rules upon which they can build happy and successful lives?

I will never forget some of my experiences in the years right after law school, when I served in juvenile court, and heard, how many times, that plaintive cry that came from a sullen youngster in his teens who said, "How was I to know better?" Nor will I forget the amazing changes that sometimes took place in such a boy if he was placed by the judge in a home with a religious background and in a different school environment.

There has been a very proper emphasis in America on the separation of church and state, a great and strong principle. But it sometimes seems as if we have carried that principle to the extent of also trying to separate our schools from the teaching of moral principles, which have for society a value and a depth and a significance that we dare not ignore, and which, though they are drawn from religion, can be taught without reference to religion, especially sectarian religion.

And so I feel—I discussed this recently with the Association of American School Administrators in their national convention—that if we are to preserve our society and retain its full depth and value, we must find a way by which we shall prepare our primary and secondary

school teachers to impart to the children the basic rules of successful moral and ethical conduct.

Of course, a recognition of the importance of moral standards has a close interrelationship with a recognition of human values and the need for better human relations, but each has a significance of its own that justifies our discussing them separately.

Currency, the third area of tension and uncertainty that I named at the outset, is a subject that may seem strange to place beside the two we have already considered, but yet it does have a direct and important place in this discussion. The means by which members of a productive free society can associate and interchange the results of their production is currency or a medium of exchange. Money, dollars, bills of exchange, are an essential part of a free and productive economy. If there is a dictatorship, a totalitarian system, the exchange can be forced by rules and laws and rationing, in some societies by compulsion so severe that death is the penalty for hoarding or for failure to live up to ration requirements, or for other evasions of economic decrees.

These various compulsions and decrees take the place of a voluntary medium of exchange by forcing the movement of goods. They do not take the place in the full and free and pleasant sense, but they do serve as a substitute means of effecting a movement of goods and the production of goods.

But if there is to be a successful maintenance of free economy, it is absolutely essential that there be a medium of exchange in which there is confidence—in other words, that a runaway inflation be not permitted.

I remember the situation in Europe right after the war. In traveling through Germany and a number of other countries, one found in various countries widely diverse degrees of stability of currency. At that particular time, early in 1947, confidence in the German mark was completely gone. There were many reasons for it, one of them, of course, being the fact that the Russians had been given a set of the engraving plates for printing marks and would just run off the marks as fast as they needed any funds, giving no accounting whatsoever. The general knowledge that the supply of marks was subject solely to whim rather than to any economic rules caused that complete lack of confidence in the mark. As a result the ration card and the permit for food in quantities determined by the kind of employment were what counted, not marks. And as a further result, goods just barely moved, and shop windows were kept empty, and workers stayed at home when they had enough marks to buy what the ration coupon for the week would allow them. The situation was truly desperate.

But the situation was corrected, and of course you know how it was done. The reform of the currency of West Germany brought out the goods and it stepped up production. The whole economy of Western Germany was changed in an amazingly short time.

Or you may remember that same period in 1947 when the money of Hungary was going through the roof. That inflation shut off production and stopped the exchange of goods.

On the other hand, consider the experience of Belgium. With remarkable foresight, while the war was still going on, the government in exile, seated in London, had a whole new set of currency printed. Then, when Allied troops liberated Belgium, the Government landed in Belgium with the new currency and immediately stopped the old currency. Balances were frozen and the new sound money began to circulate. The result was the most rapid recovery made by any western European country.

Currency, the medium of exchange, is of tremendous significance, and not only in a material sense, for nothing undermines the moral values more than a lack of respect for the means by which the fruits of toil and of saving and of endeavor can be enjoyed.

If inflation goes to a point at which individuals lose their regard for savings and developed resources and planning for the future, then not only have you destroyed so much of the immediate operating mechanism of a free economy but you have struck heavy blows against the moral fiber of the people, and the character of that fiber determines the character of their contacts one with another.

That is why I say that when we think of the great accomplishment of production and the success of the American way of life, we need to think not only of ways of working out the problems of human relationships, which are intensified by the results of economic progress, not only of ways of strengthening the moral and ethical foundations of our youth, but also of ways to prevent the deterioration of the media of economic communication, to wit, the currency of a country.

When once it has been broken loose from gold and its value depends on decisions of government alone, the problem is not an easy one and solving it will take, first of all, the aroused understanding of the consequences of inflation by the people. Such an aroused understanding ordinarily comes about, of course, after bitter experience by the people of the consequences of serious inflation. Unfortunately if that is when understanding dawns, it is rather late to work out a solution. Although smaller countries, or occupied countries, can reform their currencies and start over again, this great nation, standing in its position of world leadership and as the creditor nation of the world, could have very grave difficulties in reforming its currency after a runaway inflation.

Because of America's position in the world, inflation is a problem of the very first magnitude. For it is a very difficult disease to cure. It is a pernicious anemia in any economic system, and its consequences are disastrous. They are disastrous not only for production and the exchange of goods, but also for human relations and the moral foundations of society. And that is why I discussed a sound medium of

exchange in speaking in the memory of Edward G. Budd. For he was a man who understood human values of both the mind and the heart, and combined that understanding with great material accomplishments. Our very success in solving technical problems of invention and production has created many of the difficulties that now confront mankind. If we are to overcome these difficulties, we must find solutions to such fundamental problems as I have discussed tonight. On our success will depend whether the future accomplishments of men like Edward G. Budd will serve mankind and make life richer and fuller in the manner in which they have in generations gone by.

Let me close by saying that, though I recognize the difficulty of these problems, I nevertheless wish to sound a note of optimism. And I do so because I believe the people of America have the resources and the intelligence and the determination and the courage to meet all three, provided they are brought out for open, thoughtful consideration and examination. It is in that spirit that I have responded to your gracious invitation and have spoken so frankly and directly, and yet broadly, to you tonight, because I know that before me this evening there are so many representatives of the leadership and thoughtful citizenry of this key American community, a genuine metropolis with manifold relationships with all of the phases of life in our complex but productive and brilliant modern society.

Therefore, I invite your attention, frankly and forthrightly, to these three matters: The future relationships of men to each other, made more complex by the very consequences of our economic and productive development; the moral and ethical fiber of the future generations, given new tests and new temptations, again by the amazing developments of our economic system; and, finally, that essential of a successful economy that there be a medium of exchange in which the people have confidence, rather than of which they have fear.

If we earnestly address ourselves to the solution of the problems I have briefly sketched here, I am confident that in the years ahead we shall come upon ingenious, effective measures that will bring about their gradual solution—measures in keeping with the spirit of this hall and the spirit of him in whose memory we meet tonight, and in keeping with the great value and dignity of the human personality and its brotherhood under the fatherhood of God.

SOME EXTENSIONS OF ELEMENTARY PLASTICITY THEORY [1]

BY

F. EDELMAN [2] AND D. C. DRUCKER [3]

ABSTRACT

A detailed investigation is presented of yield or loading criteria, for work hardening materials, which lead to constant strain ratios under increasing stress when stress ratios are maintained constant. The experimenter is given a choice of incremental stress-strain theories with which results of simple or complicated combined loading tests may be correlated. These theories are dealt with in order of increasing complexity and, at the same time, increasing capacity to represent experimentally established phenomena. The objective is the use of the mathematically simplest theory which will provide the desired flexibility in the analysis of experimental data.

Loading functions of isotropic as well as anisotropic types are examined, among them those depending explicitly on stress alone and those depending on both stress and plastic strain. In the latter cases only those functions are investigated in which plastic strain appears to the first order. The predicted results of some basic tests are examined for several loading criteria.

THE USE OF TENSOR NOTATION

The behavior of materials in the plastic range is often exceedingly complicated. For this reason it is of extreme importance to prevent the basic concepts from being obscured by overly long algebraic expressions and an unnecessary number of equations. A simple understandable shorthand, called tensor notation, is therefore of great value. The understanding and use of the notation do not require any knowledge of tensors as such although elementary tensor concepts were invaluable in the original derivation of advanced stress-strain relations. All that is required are a few simple rules and a table of notation, as at the end of the paper, to avoid all need for memorization.

The familiar stress (tensor) which means σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx} to an engineer, is symbolized as σ_{ij}. Letter subscripts (except for x, y, z which have fixed directions) such as the i and j are given the values 1, 2, and 3 so that σ_{ij} stands for each term of the array

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \text{ or in engineering notation } \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}.$$

[1] The conclusions presented in this paper were obtained in the course of research conducted under Contract N7onr-358 sponsored by the Office of Naval Research and the Bureau of Ships.

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The strain components are likewise given by ϵ_{ij}.

$$\begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \equiv \begin{pmatrix} \epsilon_x & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{yx}/2 & \epsilon_y & \gamma_{yz}/2 \\ \gamma_{zx}/2 & \gamma_{zy}/2 & \epsilon_z \end{pmatrix}.$$

Here the use of $\gamma/2$ instead of γ will be recognized as the same substitution required when a Mohr's circle for strain is drawn.

Repeated letter subscripts (other than x, y, and z) in any one term mean a sum of terms. For convenience the summation sign is omitted so that for example, $\sum_{i=1}^{3}\sum_{j=1}^{3} \sigma_{ij}\epsilon_{ij}$ is written as

$$\begin{aligned} \sigma_{ij}\epsilon_{ij} = {} & \sigma_{11}\epsilon_{11} + \sigma_{22}\epsilon_{22} + \sigma_{33}\epsilon_{33} + \sigma_{12}\epsilon_{12} + \sigma_{21}\epsilon_{21} + \sigma_{23}\epsilon_{23} \\ & + \sigma_{32}\epsilon_{32} + \sigma_{31}\epsilon_{31} + \sigma_{13}\epsilon_{13} \\ \equiv {} & \sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}. \end{aligned}$$

Use of $\sigma_{ij}\epsilon_{ij}$ leaves the mind and eye uncluttered and free to really understand the fundamental ideas. Any letters may be employed for the subscripts, $\sigma_{ij}\epsilon_{ij} \equiv \sigma_{mn}\epsilon_{mn}$.

The full beauty and clarity of the notation is illustrated even better by the familiar generalized Hooke's Law. The six equations which in engineering notation are

$$\begin{aligned} \epsilon_x &= a_1\sigma_x + b_1\sigma_y + c_1\sigma_z + d_1\tau_{xy} + e_1\tau_{yz} + f_1\tau_{zx} \\ \epsilon_y &= a_2\sigma_x + b_2\sigma_y + c_2\sigma_z + d_2\tau_{xy} + e_2\tau_{yz} + f_2\tau_{zx} \\ \epsilon_z &= a_3\sigma_x + b_3\sigma_y + c_3\sigma_z + d_3\tau_{xy} + e_3\tau_{yz} + f_3\tau_{zx} \\ \gamma_{xy} &= a_4\sigma_x + b_4\sigma_y + c_4\sigma_z + d_4\tau_{xy} + e_4\tau_{yz} + f_4\tau_{zx} \\ \gamma_{yz} &= a_5\sigma_x + b_5\sigma_y + c_5\sigma_z + d_5\tau_{xy} + e_5\tau_{yz} + f_5\tau_{zx} \\ \gamma_{zx} &= a_6\sigma_x + b_6\sigma_y + c_6\sigma_z + d_6\tau_{xy} + e_6\tau_{yz} + f_6\tau_{zx} \end{aligned}$$

are all contained in the one short equation

$$\epsilon_{ij} = c_{ijkl}\sigma_{kl}$$

in tensor notation. For example, to obtain the ϵ_x equation take $i = 1$, $j = 1$ and sum over k and l from 1 to 3 because they are repeated in one term.

$$\begin{aligned} \epsilon_x \equiv \epsilon_{11} = {} & c_{1111}\sigma_{11} + c_{1122}\sigma_{22} + c_{1133}\sigma_{33} + c_{1112}\sigma_{12} + c_{1121}\sigma_{21} \\ & + c_{1123}\sigma_{23} + c_{1132}\sigma_{32} + c_{1131}\sigma_{31} + c_{1113}\sigma_{13}, \end{aligned}$$

where $c_{1111} \equiv a_1$, $c_{1122} \equiv b_1$, $c_{1133} \equiv c_1$, $c_{1112} + c_{1121} \equiv d_1$, etc.

In the following discussion, the engineering terminology and the tensor notation will be carried on in parallel for a while. The reader can become accustomed gradually to the shorthand if he is not already familiar with it. Occasional reference to the section on notation will be found necessary.

INTRODUCTION

All consistent incremental theories of plasticity are closely linked with the concept of a yield or loading function which determines whether or not, for some given set of stress increments, further plastic deformation will take place. In general, values and forms of the loading function will depend on stress, strain and their histories, although, so far, loading functions such as octahedral and maximum shearing stress, depending on stress alone, have been prevalently used in practice. In this discussion only functions in which stress or stress and strain appear explicitly will be considered.

Geometrically, at any stage of the plastic deformation the loading function may be represented by a surface in a space, whose coordinates are the components of stress. Any state of stress plots as a point in this space. Figure 1 is a familiar picture of a loading or yield surface in the

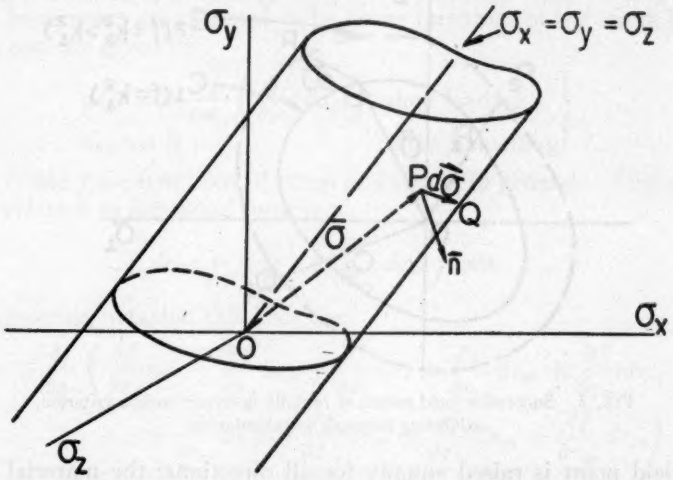

FIG. 1. Yield or loading cylinder in normal stress space.

three-dimensional normal stress space of σ_x, σ_y, σ_z, all other stress components being zero. If plastic deformation is independent of the mean normal stress $\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$, the loading surface forms a cylinder whose axis is the line $\sigma_x = \sigma_y = \sigma_z$, that is, the line whose direction cosines are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$. Additional plastic deformation will occur whenever the point P representing the existing state of stress, lies on the surface, while the point Q, representing the new state of stress, lies outside the surface. (The terminology "inside" and "outside" is not ambiguous even though the surface is not closed. "Inside" refers to that portion of the space which contains the origin, that is, the stress-

free state, and "outside" to the remainder of the space.) In other words the material is said to be loaded whenever the stress increment vector $\overline{d\sigma}$ points outward from the loading surface. "Neutral" loading is said to occur when the point Q also lies in the surface. As loading is continued the surface will, in general, move about and change its shape. The point of existing state of stress lies on the surface. In the case shown in Fig. 1 the surface will remain a cylinder with the same orientation of axis but it may translate and its cross section may change in shape.

Most loading functions in use today, apart from being initially isotropic, are of the isotropic work hardening type. According to these theories, simple tension in any direction affects the material in precisely the same manner as the same amount of tension in any other direction;

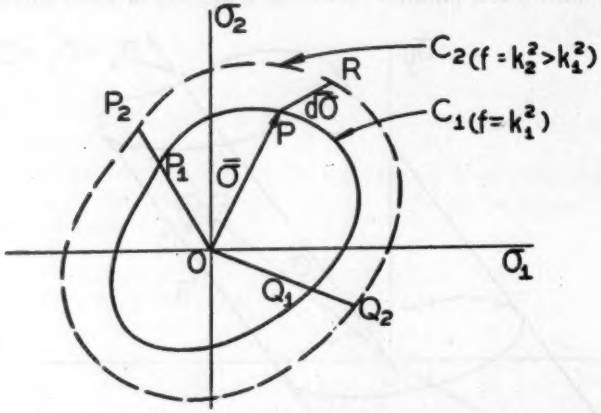

FIG. 2. Successive yield curves of initially isotropic loading criterion, exhibiting isotropic workhardening.

the yield point is raised equally for all directions; the material when unloaded is truly isotropic (1).[4] This means that the loading surface may be drawn in principal stress space, σ_1, σ_2, σ_3 replacing σ_x, σ_y, σ_z. Also the surface as a whole expands uniformly about the origin, O, during loading. Uniform expansion of the cylinder in principal stress space is equivalent to uniform expansion of any cross section about the point where it intersects the axis of the cylinder. Given any two radius vectors intersecting successive loading curves C_1 and C_2 (Fig. 2) in the (σ_1, σ_2) plane at P_1, Q_1 and P_2, Q_2 respectively, then uniform expansion implies that $OP_2/OP_1 = OQ_2/OQ_1$. As the material is loaded in biaxial tension from P to R, the loading curve changes from C_1 to C_2. As a consequence, such a theory predicts that a material will harden at equal rates both in tension and compression during any loading. In particu-

[4] The boldface numbers in parentheses refer to the references appended to this paper.

lar, if a material initially has the same yield stress in tension and compression in the σ_1-direction and if it is loaded plastically in tension in this direction to a stress σ and unloaded, then the theory predicts the new yield stress in compression in the σ_1-direction to be precisely $-\sigma$. Now it is well known that most materials, under these circumstances exhibit a compressive yield stress considerably less, numerically, than $-\sigma$. In other words, the material is softer, or at least less hardened, in compression following tension. This phenomenon, the Bauschinger effect, may often have to be taken into account by a stress-strain law if it is to be useful in practice.

BASIC RELATIONS

This paper is based on a stress strain law for work hardening materials, proposed by W. Prager (2). Tensor notation is employed for clarity and brevity; repeated subscripts denote summation. The plastic strain increments are assumed to be linear functions of the stress increments and are given by

$$\left.\begin{aligned} d\epsilon_{ij}^{\,p} &= G\frac{\partial f}{\partial \sigma_{ij}}\left(\frac{\partial f}{\partial \sigma_{pq}}d\sigma_{pq}\right) \text{ when loading} \\ d\epsilon_{ij}^{\,p} &= 0 \qquad\qquad\qquad\quad \text{when unloading} \end{aligned}\right\}, \tag{1}$$

where G and f are functions of stress and strain, in general. This stress strain relation in expanded form is

$$d\epsilon_{11}^{\,p} = G\frac{\partial f}{\partial \sigma_{11}}\left(\frac{\partial f}{\partial \sigma_{pq}}d\sigma_{pq}\right), \text{ etc.}$$

In engineering notation this becomes

$$d\epsilon_x^{\,p} = G\frac{\partial f}{\partial \sigma_x}\left(\frac{\partial f}{\partial \sigma_x}d\sigma_x + \frac{\partial f}{\partial \sigma_y}d\sigma_y + \frac{\partial f}{\partial \sigma_z}d\sigma_z + \frac{\partial f}{\partial \tau_{yz}}d\tau_{yz} + \frac{\partial f}{\partial \tau_{zy}}d\tau_{zy} + \frac{\partial f}{\partial \tau_{zx}}d\tau_{zx} + \frac{\partial f}{\partial \tau_{xz}}d\tau_{xz} + \frac{\partial f}{\partial \tau_{xy}}d\tau_{xy} + \frac{\partial f}{\partial \tau_{yx}}d\tau_{yx}\right), \text{ etc.}$$

$$d\epsilon_{12}^{\,p} = d\epsilon_{xy}^{\,p} = \tfrac{1}{2}d\gamma_{xy}^{\,p} = G\frac{\partial f}{\partial \tau_{xy}}\left(\frac{\partial f}{\partial \sigma_{pq}}d\sigma_{pq}\right), \text{ etc.}$$

The function f is written in terms of nine, rather than six, components of the stress tensor. It is symmetric in σ_{ij} and σ_{ji}, but differentiation with respect to τ_{xy}, for example, does not include differentiation with respect to τ_{yx}. For instance, writing

$$J_2 = \tfrac{1}{2}\{\tfrac{1}{3}[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2] + \tau_{yz}^2 + \tau_{zy}^2 + \tau_{zx}^2 + \tau_{xz}^2 + \tau_{xy}^2 + \tau_{yx}^2\},$$

then $\dfrac{\partial J_2}{\partial \tau_{xy}} = \tau_{xy}$. If J_2 is written in the customary fashion, making use of $\tau_{xy} = \tau_{yx}$, etc.,

$$J_2 = \tfrac{1}{6}[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2)],$$

then $\frac{\partial J_2}{\partial \tau_{xy}} = 2\tau_{xy}$, which is not the meaning intended.

When the loading function f depends on stress alone Eq. 1 may be written in the form

$$\left.\begin{aligned} d\epsilon_{ij}^{\,p} &= G\frac{\partial f}{\partial \sigma_{ij}}df \qquad df \geqslant 0 \\ d\epsilon_{ij}^{\,p} &= 0 \qquad\qquad\quad df \leqslant 0 \end{aligned}\right\}. \tag{2}$$

Total strain increments $d\epsilon_{ij} = d\epsilon_{ij}^{\,e} + d\epsilon_{ij}^{\,p}$, where $d\epsilon_{ij}^{\,e}$ are the elastic strain increments given by Hooke's law.

It is the purpose of this paper to start with the very simplest loading functions and consider more elaborate forms up to a stage at which the complexity of the loading function outweighs its usefulness at the present time. Throughout the following, whenever f depends on strain as well as stress, plastic strains will appear to the first order only. All but the last two functions examined here are initially isotropic, that is, anisotropy of the loading function, if any, is introduced by the plastic deformation.

This extension of presently employed theories preserves many of their concepts. In particular, the following two assumptions are made as being quite good approximations for reported experimental data on solid bars and thin-walled tubes:

(1) Radial loading, that is, loading with constant ratios between components of stress, $\sigma_{ij} = K\sigma_{ij}^{\,o}$ produces constant plastic strain ratios. This means that the ratios $\epsilon_{ij}^{\,p}/\epsilon_{kl}^{\,p}$ are independent of the monotonely increasing loading parameter K. From this it follows at once that the ratios of the partial derivatives of f with respect to stress, $\frac{\partial f}{\partial \sigma_{ij}}\Big/\frac{\partial f}{\partial \sigma_{kl}}$, must be independent of K. This assumption is made so that, for radial loading, the μ *vs.* ν curve (Lode's variables), which is determined by the loading function f, will not depend upon K (3). To each $\mu = \dfrac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \dfrac{3s_2}{s_1 - s_3}$, where s_1, s_2, s_3 are the principal stress deviations, there will correspond one and only one value of $\nu = \dfrac{2d\epsilon_2^{\,p} - d\epsilon_1^{\,p} - d\epsilon_3^{\,p}}{d\epsilon_1^{\,p} - d\epsilon_3^{\,p}} = \dfrac{3d\epsilon_2^{\,p}}{d\epsilon_1^{\,p} - d\epsilon_3^{\,p}}$, where $d\epsilon_1^{\,p}$, $d\epsilon_2^{\,p}$, $d\epsilon_3^{\,p}$ are the principal plastic strain increments (4).

(2) No plastic volume changes occur, that is, the sum of the normal strain increments $d\epsilon_{kk}^{\,p} \equiv d\epsilon_x^{\,p} + d\epsilon_y^{\,p} + d\epsilon_z^{\,p}$ and therefore $\frac{\partial f}{\partial \sigma_{kk}} \equiv \frac{\partial f}{\partial \sigma_x} + \frac{\partial f}{\partial \sigma_y} + \frac{\partial f}{\partial \sigma_z}$ is zero. Thus f, although often more conveniently written in terms of the stress σ_{ij}, actually depends on the stress deviator s_{ij} alone,

since, if $p = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$ denotes the mean normal stress, $\frac{\partial f}{\partial \sigma_{kk}} = 0$ implies $\frac{\partial f}{\partial p} = 0$. In engineering terminology, the loading function depends on the shear stresses only. It is independent of the mean normal stress.

LOADING FUNCTION AND STRESS-STRAIN LAWS

In the following analysis an ideal thin-walled circular cylinder will be considered so that the stresses may be conveniently visualized and actually attained in practice. Several tests will be applied to illustrate the predictions of some of the theories. The results of each will be discussed in detail; conversely, if actual tests are performed on tubes, a comparison of the experimental results with the predictions will indicate the most suitable theory to be used. The tests are:

(1) Initial tension in the x (axial) direction, producing $\epsilon_x^p = \epsilon$ and unloading followed by: (*a*) compression in the x (axial) direction; or (*b*) tension in the y (circumferential) direction; or (*c*) torsion about the x-axis, producing constant shear stress on the cross section in the circumferential direction.

(2) Initial torsion about the x-axis or simple shear, producing $\gamma_{xy}^p = \gamma$ and unloading followed by: (*a*) reversed torsion about the x-axis, or (*b*) tension in the x-direction.

I. $f = J_2$

The simplest law

$$d\epsilon_{ij}^p = G(J_2)s_{ij}dJ_2 \qquad dJ_2 \geqslant 0, \tag{3}$$

or, in engineering notation,

$$d\epsilon_x^p = \tfrac{1}{3}G(J_2)[2\sigma_x - \sigma_y - \sigma_z]dJ_2, \text{ etc.}$$

$$d\gamma_{xy}^p = 2G(J_2)\tau_{xy}dJ_2, \text{ etc.},$$

results from the loading criterion of von Mises, namely, $f = J_2$ which is equivalent to the octahedral shear stress criterion. It can be seen at once from the definitions of μ and ν that, since the plastic strain increments are proportional to the stress deviator s_{ij}, $\mu = \nu$ for every radial loading. This does not agree with experimental fact for most engineering metals (3). However, this stress-strain law does give constancy of strain ratios and zero plastic volume change.

For a state of biaxial stress, with principal stresses σ_1, σ_2, $J_2 = \frac{1}{3}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2) = k^2$ represents a family of ellipses with parameter k in the (σ_1, σ_2) plane, whose major axes are inclined to the σ_1-axis at 45° and whose centers coincide with the origin. In Fig. 3, the solid curve ($k = k_1$) is the initial yield curve and the dotted one ($k = k_2 > k_1$)

is a new yield curve after some plastic deformation has taken place. This J_2-theory contains not only the aforementioned initial isotropy ($OB = OC$ and $OA = OD$) and isotropic work hardening ($OB' = OC'$ and $OA' = OD'$) but also equality of tensile and compressive yield stresses ($OA' = OB'$) at all stages of the deformation. The theory, stating equal hardening in tension and compression, takes no account of the Bauschinger effect.

Prestressing in tension or shear merely expands a J_2 loading curve. Subsequent tension or compression in any direction are equivalent in this simple theory which cannot take into account any cross effects. Uniaxial stress produces a normal plastic strain increment $\Delta\epsilon$ in its own direction and $-\Delta\epsilon/2$ at right angles. No shearing action is predicted, and conversely, shear stress alone produces no normal strain at any stage of the deformation.

Thus, as could be expected, the extreme mathematical simplicity of the loading function $f = J_2$ or $f(J_2)$ (there is no distinction here) allows

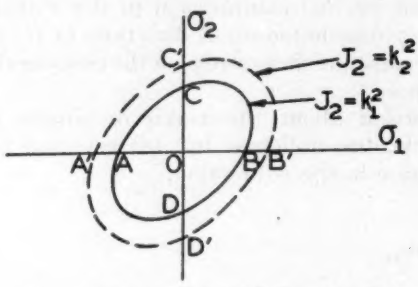

FIG. 3. Successive yield curves of $f = J_2$.

us to fit only limited experimental data well. A plot of octahedral shearing stress *vs.* octahedral shearing strain for radial loading is a good example (5). The incorrectness of the relation $\mu = \nu$ and the absence of the Bauschinger effect can be remedied only by making f more complex.

II. $f = f(J_2, J_3)$

The natural extension of $f = f(J_2)$, namely, $f = f(J_2, J_3)$, arises from the fact that all stress invariants can be expressed in terms of the sum of the principal stresses and the second and third stress invariants, J_2 and J_3. Unlike J_2, which represents shear strain energy, $J_3 \equiv \frac{1}{27}(2\sigma_1 - \sigma_2 - \sigma_3)(2\sigma_2 - \sigma_1 - \sigma_3)(2\sigma_3 - \sigma_1 - \sigma_2)$ has no simple physical interpretation. It is a weighting factor for the shear stresses. The simplest form of $f(J_2, J_3)$ which satisfies the assumption of constant plastic strain ratios under radial loading is

$$f = J_2^3 - cJ_3^2. \tag{4}$$

It is homogeneous in the stresses and thus the ratios $\partial f/\sigma_{ij}/\partial f/\partial\sigma_{pq}$ and therefore the strain ratios will be independent of K. As in the previous section no more generality is achieved by writing $f = f(J_2^3 - cJ_3^2)$. The function $f = J_2^3 - cJ_3^2$ plotted in the (σ_1, σ_2) plane represents a family of curves symmetric about the lines $\sigma_1 = \pm\sigma_2$, thus predicting initial isotropy and equality of yield stresses in tension and compression. The latter can be seen to be due to the fact that this particular f is an even function of stress, that is, $f(\sigma) = f(-\sigma)$. As in the preceding section the function $f(J_2, J_3)$ contains isotropic work hardening, that is, no Bauschinger effect. The yield curve in the (σ_1, σ_2) plane expands uniformly about the origin as plastic deformation takes place. Figure 4

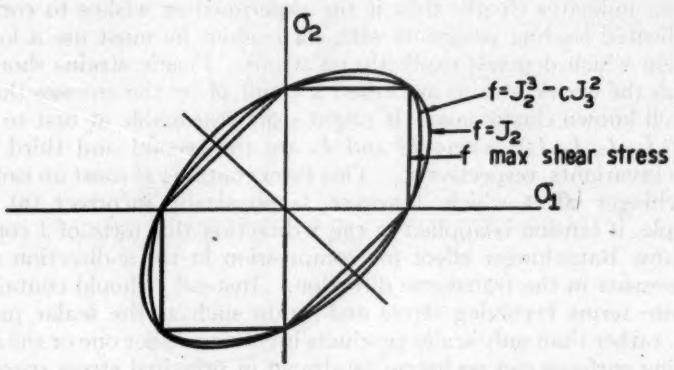

FIG. 4. The yield criterion $f = J_2^3 - cJ_3^2$ in relation to those of octahedral and maximum shear stress.

shows a plot of $J_2^3 - cJ_3^2 = k^2$ which lies between the ellipse of $J_2 = k^2$ and the inscribed hexagon of the maximum shear stress criterion. This law no longer predicts $\mu = \nu$, since $d\epsilon_{ij}^p$ is not proportional to s_{ij}. It often allows us to fit the experimental μ *vs.* ν curve by suitable choice of the constant c, although in general a more elaborate form of $f(J_2, J_3)$ is required for complete correlation with experiments on tubes.

The stress-strain law resulting from the general form $f = f(J_2, J_3)$ is

$$d\epsilon_{ij}^p = G(J_2, J_3)\left[\frac{\partial f}{\partial J_2}(J_2, J_3)s_{ij} + \frac{\partial f}{\partial J_3}(J_2, J_3)t_{ij}\right]df(J_2, J_3) \qquad (5)$$

$$df \geqslant 0.$$

Clearly if the plastic strain ratios are to be constant for $\sigma_{ij} = K\sigma_{ij}^0$, f must be homogeneous in stress so that $\dfrac{\partial f}{\partial J_2}$ is of degree one higher in stress than $\dfrac{\partial f}{\partial J_3}$. Then the tensor appearing in the square bracket of

Eq. 5 is homogeneous in stress. The assumption of incompressibility is satisfied since s_{ij} and t_{ij} are deviatoric tensors, that is, $s_{kk} = t_{kk} = 0$.

To sum up, a (J_2, J_3) theory will permit exact fitting of test results for simple (radial loading) tube tests, as it will fit any μ *vs.* ν relationship. Although it cannot take into account any Bauschinger or allied effect, the experimental analyst may find this theory sufficient also for more complicated loading paths, as long as they are confined to a sufficiently narrow angular region in the (σ_1, σ_2) plane, such as one covering part of the tension-tension quadrant, for instance.

III. $f = F(J_2) - ms_{ij}\epsilon_{ij}^p = F(J_2) - m\sigma_{ij}\epsilon_{ij}^p$

The absence of the Bauschinger effect in the J_2 as well as the J_2, J_3 theories indicates clearly that if the experimenter wishes to correlate complicated loading programs with each other, he must use a loading function which depends explicitly on strain. Plastic strains should be used as the elastic strains are taken account of by the stresses through the well known elastic law. It might seem reasonable at first to write $f = f(J_2, J_3, I_2, I_3)$, where I_2 and I_3 are the second and third order strain invariants, respectively. This form contains at most an isotropic Bauschinger effect which, however, is physically incorrect (6). For example, if tension is applied in the x-direction this form of f contains the same Bauschinger effect for compression in the x-direction as for compression in the transverse direction. Instead f should contain one or more terms involving stress *and* strain such as the scalar product $s_{ij}\epsilon_{ij}^p$, rather than only scalar products involving either one or the other. Loading surfaces can no longer be drawn in principal stress space; the coordinates must be the components of stress for axes fixed in the material. Attention is drawn again to the restriction that only first order strain theories will be dealt with here.

The simplest extension of the J_2 theory which exhibits a reasonable Bauschinger effect is

$$f = J_2 - ms_{ij}\epsilon_{ij}^p = J_2 - m\sigma_{ij}\epsilon_{ij}^p \qquad (6)$$

(as ϵ_{ij}^p is a deviator), where m is a constant. This law, however, reintroduces the drawback of $f = J_2$ in that it leads to the relation $\mu = \nu$ which is seldom found in experiments. For radial loading the ratios

$$\frac{d\epsilon_{ij}^p}{d\epsilon_{kl}^p} = \frac{\epsilon_{ij}^p}{\epsilon_{kl}^p} = \frac{s_{ij} - m\epsilon_{ij}^p}{s_{kl} - m\epsilon_{kl}^p} = \frac{s_{ij}}{s_{kl}}$$

are constant, and hence, $\mu = \nu$. In Fig. 5, a plot of σ_x *vs.* σ_y with all other stress components zero, the solid curve, $f = k^2$, is for initial yield. This curve is obviously identical with that in Fig. 3, since $f = J_2$, when $\epsilon_{ij}^p = 0$. The other curves are plots of $f = k^2 > k_1^2$, for various values of k after the material has been loaded in tension to the point B'. They remain ellipses since $f = k^2$ is still of second degree in stress. Further-

more their orientations have remained unchanged. Their centers are points on the σ_x-axis. Finally the original ellipse has dilated an amount determined by the increase in the value of the loading function.

Thus this simple theory allows for the experimentally well established fact that a tensile stress σ_x produces a yield point in tension which is higher than that in compression in the same direction. For sufficiently small dilatation of the yield curve, this theory may account for either hardening (curve 4) or actual softening (curves 2 and 3) in compression following tension. In any case pre-tension will produce more hardening in tension than in compression, $OB' > OA_n$. For the transverse direc-

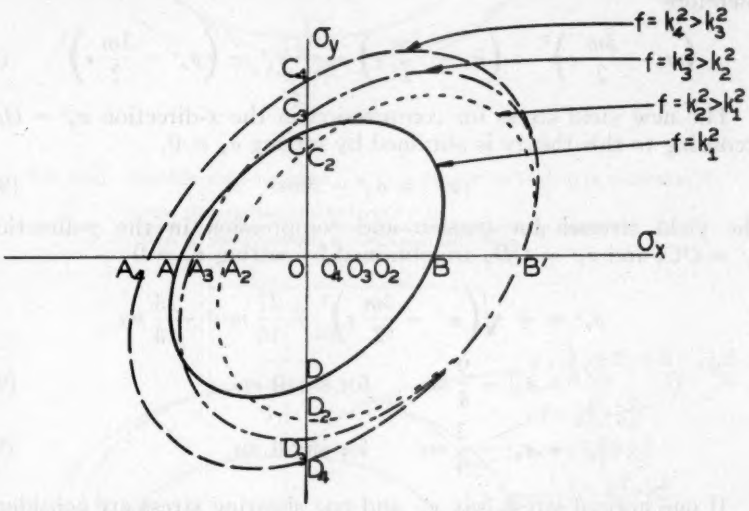

FIG. 5. Possible yield curves of $f = J_2 - ms_{ij}\epsilon_{ij}^p$ for loading in axial tension (σ_x) to B'.

curve (1) is the initial yield curve $f = J_2$,
curve (2) shows softening in compression and transverse (σ_y) tension,
curve (3) shows softening in compression and hardening in transverse tension,
curve (4) shows hardening in compression and transverse tension.

tion, which has experienced a contraction in longitudinal pre-tension, the theory predicts a higher yield point in subsequent compression than in tension, $OD_n > OC_n$. Again the transverse direction may soften in tension (curve 2) following longitudinal tension for sufficiently small $k_2 - k_1$.

It is of interest and importance in analyzing experimental data to have the algebraic expressions given by the theory for the stresses at which the material will yield in tension and compression in the x and y directions after having been loaded in tension to B' (σ_x having there the value σ_x') and unloaded.

Considering stresses σ_x and σ_y alone, $J_2 = \frac{1}{3}(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2)$ and

$s_{ij}\epsilon_{ij}^{p} = \sigma_{ij}\epsilon_{ij}^{p} = \sigma_x\epsilon_x^{p} + \sigma_y\epsilon_y^{p} = \sigma_x\epsilon - \sigma_y\frac{\epsilon}{2}$ for the loading path OB'. ϵ is the plastic strain produced by $\sigma_x{}^t$. The loading function $J_2 - m\sigma_{ij}\epsilon_{ij}^{p}$ at this stage is

$$f = \tfrac{1}{3}(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2) - m\sigma_x\epsilon + \frac{m}{2}\sigma_y\epsilon = k_2^2$$

or

$$\frac{1}{3}\left[\left(\sigma_x - \frac{3m}{2}\epsilon\right)^2 - \left(\sigma_x - \frac{3m}{2}\epsilon\right)\sigma_y + \sigma_y^2\right] = k_2^2 + \frac{3}{4}m^2\epsilon^2. \tag{7}$$

Therefore

$$\left(\sigma_x - \frac{3m}{2}\epsilon\right)^2 - \left(\sigma_x - \frac{3m}{2}\epsilon\right)\sigma_y + \sigma_y^2 = \left(\sigma_x{}^t - \frac{3m}{2}\epsilon\right)^2. \tag{8}$$

The new yield stress for compression in the x-direction $\sigma_x{}^c = OA_n$ according to this theory is obtained by setting $\sigma_y = 0$,

$$|\sigma_x{}^c| = \sigma_x{}^t - 3m\epsilon. \tag{9a}$$

The yield stresses for tension and compression in the y-direction, $\sigma_y{}^t = OC_n$ and $\sigma_y{}^c = OD_n$ are obtained by setting $\sigma_x = 0$

$$\sigma_y{}^t = +\sqrt{\left(\sigma_x{}^t - \frac{3m}{2}\epsilon\right)^2 - \frac{27}{16}m^2\epsilon^2} - \frac{3}{4}m\epsilon$$

$$\doteq \sigma_x{}^t - \frac{9}{4}m\epsilon \qquad \text{for small } m\epsilon \tag{9b}$$

$$|\sigma_y{}^c| \doteq \sigma_x{}^t - \frac{3}{4}m\epsilon \qquad \text{for small } m\epsilon. \tag{9c}$$

If one normal stress, say σ_x, and one shearing stress are considered

$$\begin{aligned} f &= \frac{1}{3}\left(\sigma_x^2 + \frac{3}{2}\tau_{xy}^2 + \frac{3}{2}\tau_{yx}^2\right) - m\sigma_x\epsilon_x - m\tau_{xy}\epsilon_{xy} - m\tau_{yx}\epsilon_{yx} \\ &= \frac{1}{3}(\sigma_x^2 + 3\tau_{xy}^2) - m\sigma_x\epsilon_x - m\tau_{xy}\gamma_{xy} \end{aligned} \tag{10}$$

in the usual engineering notation. The initial yield and subsequent loading ellipses are shown in Fig. 6(a) for tensile loading, Fig. 6(b) for shear. As the axes of the initial yield ellipse are the coordinate axes σ_x, τ_{xy} there is much less true cross effect than for Fig. 5. If, for example, the ellipse should not dilate, to a first approximation, pre-tension would not change the yield point in shear and conversely. For initial tension to B'

$$f = \frac{1}{3}(\sigma_x^2 + 3\tau_{xy}^2) - m\sigma_x\epsilon = \frac{1}{3}\left[\left(\sigma_x{}^t - \frac{3m}{2}\epsilon\right)^2 - \frac{9}{4}m^2\epsilon^2\right]$$

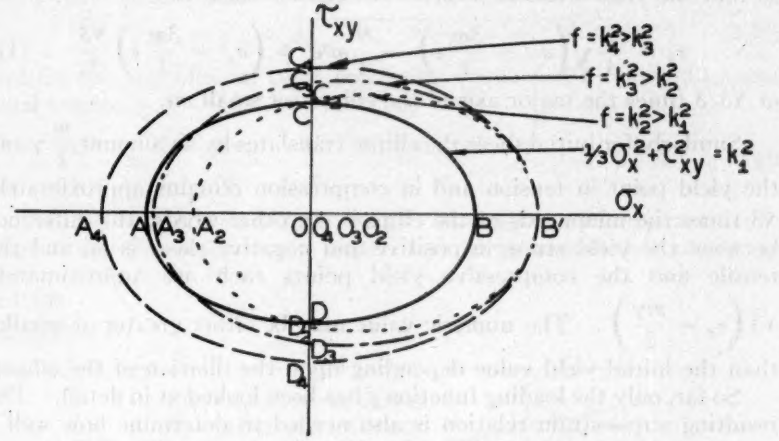

FIG. 6(*a*). Possible yield curves of $f = J_2 - ms_{ij}e_{ij}^p$ for loading in tension to B'.

curve (1) is the original yield curve $f = J_2$,
curve (3) shows softening in compression and hardening in torsion,
curve (4) shows hardening in compression and torsion.

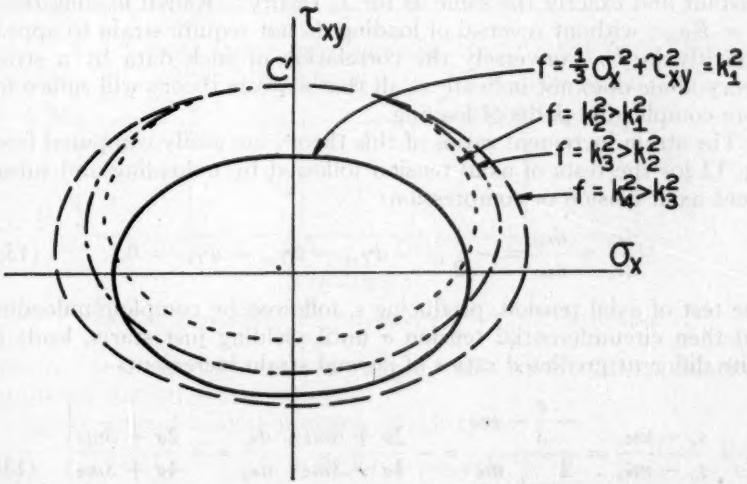

FIG. 6(*b*). Possible yield curves of $f = J_2 - ms_{ij}e_{ij}$ for loading in torsion to C'.

curve (1) is the original yield curve $f = J_2$,
curve (2) shows softening in reverse torsion and in tension and compression,
curve (3) shows softening in reverse torsion and hardening in tension and compression,
curve (4) shows hardening in reverse torsion and in tension and compression.

so that the yield stress in positive or negative shear is

$$\tau_y = \frac{\sqrt{3}}{3}\sqrt{\left(\sigma_x' - \frac{3m}{2}\epsilon\right)^2 - \frac{9}{4}m^2\epsilon^2} \doteq \left(\sigma_x' - \frac{3m}{2}\epsilon\right)\frac{\sqrt{3}}{3} \qquad (11)$$

or $\sqrt{3}/3$ times the major axis of the ellipse for small $m\epsilon$.

Similarly for initial shear the ellipse translates by an amount $\frac{m}{2}\gamma$ and the yield point in tension and in compression remains approximately $\sqrt{3}$ times the minor axis of the ellipse. In other words, the difference between the yield stress in positive and negative shear is m, and the tensile and the compressive yield points each are approximately $\sqrt{3}\left(\tau_y - \frac{m\gamma}{2}\right)$. The numeric value may be either greater or smaller than the initial yield value depending upon the dilation of the ellipse.

So far, only the loading function f has been looked at in detail. The resulting stress-strain relation is also needed to determine how well a set of experimental data will be fitted by the theory. From Eq. 1

$$d\epsilon_{ij}^{p} = G(s_{ij} - m\epsilon_{ij}^{p})\frac{\partial f}{\partial \sigma_{pq}}d\sigma_{pq}. \qquad (12)$$

At $\epsilon_{ij}^{p} = 0$, and at all ϵ_{ij}^{p} for radial loading, the plastic strain ratios are constant and exactly the same as for J_2 theory. Radial loading tests, $\sigma_{ij} = K\sigma_{ij}^{\circ}$, without reversal of loading do not require strain to appear explicitly in f. Conversely the correlation of such data by a stress theory alone does not indicate at all that a stress theory will suffice for more complicated paths of loading.

The strain increment ratios of this theory are easily computed from Eq. 12 for the tests of axial tension followed by unloading and subsequent axial tension or compression:

$$\frac{d\epsilon_y}{d\epsilon_x} = \frac{d\epsilon_z}{d\epsilon_x} = -\frac{1}{2}, \qquad d\gamma_{xy} = d\gamma_{yz} = d\gamma_{zx} = 0. \qquad (13a)$$

The test of axial tension, producing ϵ, followed by complete unloading and then circumferential tension σ until yielding just starts, leads to quite different predicted ratios of normal strain increments.

$$\left.\begin{aligned} \frac{d\epsilon_x}{d\epsilon_y} = \frac{s_x - m\epsilon_x}{s_y - m\epsilon_y} = \frac{-\frac{\sigma}{3} - m\epsilon}{\frac{2}{3}\sigma + \frac{m\epsilon}{2}} = -\frac{2\sigma + 6m\epsilon}{4\sigma + 3m\epsilon}, \quad \frac{d\epsilon_z}{d\epsilon_y} = -\frac{2\sigma - 3m\epsilon}{4\sigma + 3m\epsilon} \\ d\gamma_{xy} = d\gamma_{yz} = d\gamma_{zx} = 0, \end{aligned}\right\} \qquad (13b)$$

for the test of axial tension followed by unloading and axial twist τ:

$$\frac{d\epsilon_y}{d\epsilon_x} = \frac{d\epsilon_z}{d\epsilon_x} = -\frac{1}{2}, \qquad \frac{d\epsilon_x}{d\gamma_{xy}} = \frac{d\epsilon_z}{2d\epsilon_{xy}} = -2\tau, \qquad d\gamma_{zx} = d\gamma_{yz} = 0 \qquad (13c)$$

for the test of axial twist and continued or reversed axial twist:

$$d\epsilon_x = d\epsilon_y = d\epsilon_z = 0, \qquad d\gamma_{zx} = d\gamma_{yz} = 0, \tag{13d}$$

and for the test of axial twist producing γ followed by unloading and axial tension σ:

$$\frac{d\epsilon_y}{d\epsilon_x} = \frac{d\epsilon_z}{d\epsilon_x} = -\frac{1}{2}, \quad \frac{d\gamma_{xy}}{d\epsilon_x} = \frac{2d\epsilon_{xy}}{d\epsilon_x} = -\frac{3m\gamma}{\sigma}, \quad d\gamma_{zx} = d\gamma_{yz} = 0. \tag{13e}$$

As in this relatively simple theory there is only one arbitrary constant at our disposal, namely, m, we have no guarantee that, once m is

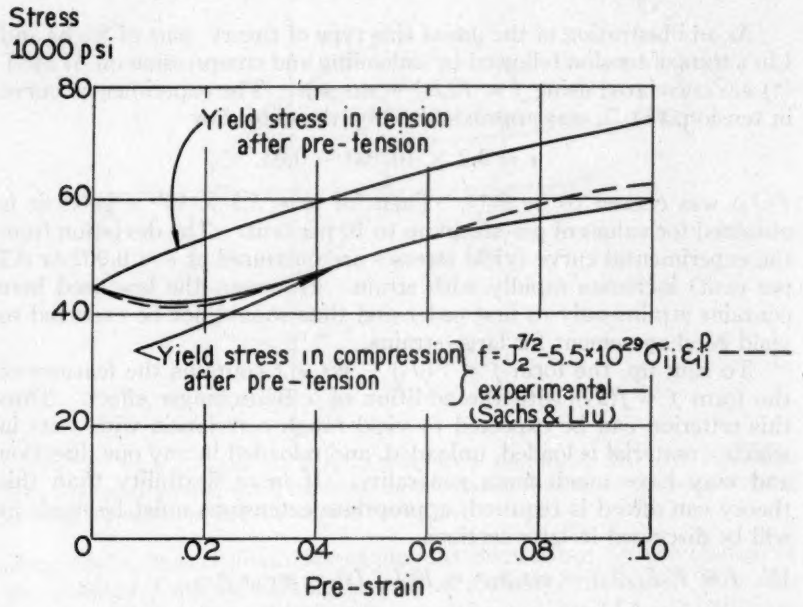

FIG. 7. Correlation of test data using $f = J_2^{7/2} - 5.5 \times 10^{29}\sigma_{ij}\epsilon_{ij}^p$.

fitted to one particular test, the theory will agree with experimental results for the remaining data.

In the general loading function of this type

$$f = F(J_2) - ms_{ij}\epsilon_{ij}^p \tag{14}$$

a form of F can be chosen to fit data somewhat better than $F = J_2$. Additional flexibility may be obtained by including some arbitrary constant, such as in $F(J_2) = J_2^2 + aJ_2$ for example. It is to be noted here that the equivalent generalization of $f = J_2$, Sec. I, to $f = f(J_2)$ provides no extension since the factor $(df/dJ_2)^2$ can be absorbed in the function G, Eq. 1. The corresponding results for the yield stresses and

strain increment ratios can be computed precisely as those above. A fairly simple choice to handle is the linear form

$$f = \sqrt{J_2} - ms_{ij}\epsilon_{ij}^p = \sqrt{J_2} - m\sigma_{ij}\epsilon_{ij}^p. \tag{15}$$

This loading function leads to the yield stress ratios following simple tension σ_z^t:

$$\frac{|\sigma_z^c|}{\sigma_z^t} = \frac{p}{p + 2m\epsilon}, \qquad \frac{\sigma_y^t}{\sigma_z^t} = \frac{p}{p + \frac{3}{2}m\epsilon}, \qquad \frac{|\sigma_y^c|}{\sigma_z^t} = \frac{p}{p + \frac{1}{2}m\epsilon}, \tag{16}$$

where $p = \dfrac{1}{\sqrt{3}} - m\epsilon$.

As an illustration of the use of this type of theory, part of Sachs and Liu's tests of tension followed by unloading and compression on Al 24ST (7) are correlated using $f = F(J_2) - ms_{ij}\epsilon_{ij}^p$. The experimental curve in tension, Fig. 7, was approximated by the cubic law

$$\epsilon = 3.2 \times 10^{-16}\sigma^3 - 0.03.$$

$F(J_2)$ was chosen to be $J_2^{7/2}$. Then for $m = 5.5 \times 10^{20}$ a good fit is obtained for values of pre-strain up to 10 per cent. The deviation from the experimental curve (yield stresses are measured at $\epsilon = 0.002$ or 0.2 per cent) increases rapidly with strain. However, the law used here contains strains only to first order and thus should not be expected to yield good agreement for large strains.

To sum up, the form $f = F(J_2) - ms_{ij}\epsilon_{ij}^p$ contains the features of the form $f = f(J_2)$ with the addition of a Bauschinger effect. Thus this criterion can be expected to yield rough correlation with tests in which a material is loaded, unloaded, and reloaded in any one direction and may have much more generality. If more flexibility than this theory can afford is required, appropriate extensions must be made as will be discussed in later sections.

IV. $f = F(J_2, J_3) - ms_{ij}\epsilon_{ij}^p = F(J_2, J_3) - m\sigma_{ij}\epsilon_{ij}^p$

The next elaboration of the loading function is now obvious. It was seen in Sec. II that the inclusion of J_3 in the loading function can produce the proper μ *vs.* ν relationship but gave no Bauschinger effect, while Sec. III showed that introducing strains into f resulted in a Bauschinger effect but also gave the undesirable relation $\mu = \nu$. The desirable features of the two can be combined directly by writing

$$f = F(J_2, J_3) - ms_{ij}\epsilon_{ij}^p, \tag{17}$$

where F need not be homogeneous in the stresses themselves but must be a function of a homogeneous function such as $F(J_2, J_3) = (J_2^3 - cJ_3^2)^2 + a(J_2^3 - cJ_3^2)$, for example, so that the ratios $\dfrac{\partial f}{\partial \sigma_{ij}} \Big/ \dfrac{\partial f}{\partial \sigma_{kl}}$ will be con-

stant as K increases in $\sigma_{ij} = K\sigma_{ij}^{\circ}$. As in the previous section the inclusion of the arbitrary constant a will increase the flexibility of the theory.

As before, the solid curve in Fig. 8 is the plot of the initial yield curve $f = F = k_1^2$ in the (σ_x, σ_y) plane. Loading in tension in the x-direction to the point B' distorts the initial yield curve into the dotted curve, $f = k_2^2 > k_1^2$. This distortion now consists of a translation

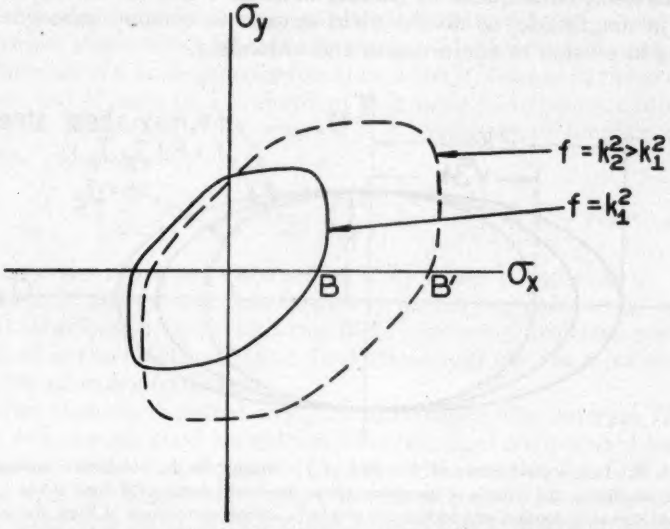

FIG. 8. Yield curves of $f = F(J_2, J_3) - ms_{ij}\epsilon_{ij}^{p}$ in axial tension (σ_x) to B'. The solid curve is the initial yield curve and the dotted curve is a possible subsequent yield curve showing softening in compression and in transverse tension (σ_y).

along the σ_x-axis, a dilatation of amount determined by the change of the value of f during loading as well as a change of shape. The significant yield stress ratios for the special case $F = (J_2^3 - cJ_3^2)^{1/6}$ are computed to be

$$\frac{|\sigma_x^c|}{\sigma_x^t} = \frac{p}{p + 2m\epsilon}, \qquad \frac{\sigma_y^t}{\sigma_x^t} = \frac{p}{p + \frac{3}{2}m\epsilon}, \qquad \frac{|\sigma_y^c|}{\sigma_x^t} = \frac{p}{p + \frac{1}{2}m\epsilon},$$
$$p = \left[\frac{1}{27} - \frac{4c}{(27)^2}\right]^{1/6} - m\epsilon, \tag{18}$$

a form similar to Eq. 16. The exponent $\frac{1}{6}$ is chosen to reduce F to first order in stress. Any other exponent could, of course, have been chosen or, for that matter, any other function of the form specified above. Simple yield stress ratios will not generally result but better agreement with experiment is possible.

In the (σ, τ) plane, Fig. 9, of combined tension-torsion the criteria of maximum and of octahedral shear stress are represented by ellipses, centered at the origin, whose major axes (coinciding with the σ-axis) have lengths $2k$ and $\sqrt{3}k$, respectively, when the minor axis has length k. The initial yield curve of the (J_2, J_3) criterion will usually lie between the two ellipses, and will not be an ellipse itself but will roughly resemble one in shape. As in the previous section the tensile and compressive yield stresses, subsequent to loading in torsion and unloading, remain equal in magnitude, as do the yield stresses in torsion, subsequent to loading in tension or compression and unloading.

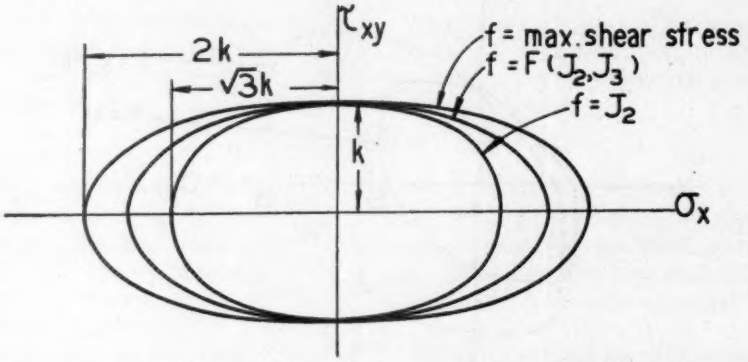

FIG. 9. Initial yield curve of $f = F(J_2, J_3) - ms_{ij}\epsilon_{ij}^p$ in the combined tension-torsion plane in relation to the criteria of maximum shear stress and octahedral shear stress. Successive yield curves in tension and torsion are precisely analogous to those of Figs. 6(*a*) and 6(*b*).

The stress-strain relation corresponding to the loading function $f = F(J_2, J_3) - ms_{ij}\epsilon_{ij}^p = F(J_2, J_3) - m\sigma_{ij}\epsilon_{ij}^p$ is given by Eq. 1 as

$$d\epsilon_{ij}^p = G(J_2, J_3)\left[\frac{\partial F}{\partial J_2}s_{ij} + \frac{\partial F}{\partial J_3}t_{ij} - m\epsilon_{ij}^p\right]\left(\frac{\partial f}{\partial \sigma_{pq}}d\sigma_{pq}\right). \quad (19)$$

Therefore, just as for $f = f(J_2, J_3)$, the ratios

$$\frac{d\epsilon_{ij}^p}{d\epsilon_{kl}^p} = \frac{\epsilon_{ij}^p}{\epsilon_{kl}^p} = \frac{\dfrac{\partial F}{\partial J_2}s_{ij} + \dfrac{\partial F}{\partial J_3}t_{ij} - m\epsilon_{ij}^p}{\dfrac{\partial F}{\partial J_2}s_{kl} + \dfrac{\partial F}{\partial J_3}t_{kl} - m\epsilon_{kl}^p} = \frac{\dfrac{\partial F}{\partial J_2}s_{ij} + \dfrac{\partial F}{\partial J_3}t_{ij}}{\dfrac{\partial F}{\partial J_2}s_{kl} + \dfrac{\partial F}{\partial J_3}t_{kl}} \quad (20)$$

are constants as K increases in $\sigma_{ij} = K\sigma_{ij}^e$ whenever F is a function of a homogeneous function of the stresses. The computation of the ratios is simple and direct for any chosen F.

To sum up, the general form $f = F(J_2, J_3) - ms_{ij}\epsilon_{ij}^p$ contains a Bauschinger effect but its flexibility in this respect is again limited as there is only one constant, m, in the strain term. As in the case of

$f = f(J_2, J_3)$ this theory permits correlation of tube tests and, possibly, of complicated tests whose loading paths are confined to a sufficiently narrow angular region in, say, the (σ_x, σ_y) plane. Thus in this form the best features of $f = f(J_2, J_3)$ and $f = F(J_2) - ms_{ij}\epsilon_{ij}^p$ are combined.

V. $f = F(J_2, J_3) - H(J_2, J_3)s_{ij}\epsilon_{ij}^p$

More flexibility is achieved at once by substituting a function H for the single constant m in the expression for the loading function. However, this additional freedom is somewhat restricted by the assumption of constant strain ratios for radial loading. The function F must again be a function of a homogeneous function of the stresses as in the previous section, and H must be a function of that same homogeneous function. Thus $F = F(h)$, $H = H(h)$, where h is a homogeneous function of the stresses. Then we have

$$\frac{\partial f}{\partial \sigma_{ij}} = \left[\frac{dF}{dh} - (s_{pq}\epsilon_{pq}^p)\frac{dH}{dh}\right]\left(\frac{\partial h}{\partial J_2}s_{ij} + \frac{\partial h}{\partial J_3}t_{ij}\right) - H\epsilon_{ij}^p \qquad (21)$$

and hence the ratios are constant as K increases in $\sigma_{ij} = K\sigma_{ij}^o$. The experimental analyst will thus be able to match one point of an experimental Bauschinger curve with this theory for every arbitrary constant contained in the function $H(h)$. In particular, H may be a polynomial in h with arbitrary coefficients.

It can then be expected that, by introducing the function H, the theory will provide good correlation with results of complicated loading tests, even though the choice of H is somewhat restricted. Still better control over the Bauschinger effect would, of course, be achieved by a law in which this function H is quite arbitrary as in the next section.

VI. $f = F(J_2, J_3) - [p(J_2, J_3)s_{ij} + q(J_2, J_3)t_{ij}]\epsilon_{ij}^p$

Again F must be a function of a homogeneous function in the stresses, that is, $F = F(h)$. In addition the assumption of constant strain ratios during radial loading imposes a condition on the function q. Whereas in the previous theory, Sec. V, the function p was restricted to depend on h only, it is now quite arbitrary. The function q must satisfy a simple linear differential equation to compensate, so to speak, for the arbitrary character of p. It can then be expected that this function will furnish excellent control over the Bauschinger and cross effects.

Comparison with Sec. III, dealing with $f = J_2 - ms_{ij}\epsilon_{ij}^p$, shows that we have paid for the flexibility of the present law with a marked increase in complexity. This may, in many cases, render the law disadvantageous. Nevertheless, at this stage we have arrived at a law which will enable the experimental analyst to correlate results of many loading and reverse loading tests.

It was not thought worthwhile at present to introduce the term containing the deviator of the product tensor $t_{ik}\epsilon_{kj}^p$ into the expression

for $\frac{\partial f}{\partial \sigma_{ij}}$, although the restrictions on its coefficient may be readily found. The addition of this term would complete the first order strain theories for initially isotropic material.

VII. $f = \frac{1}{2}c_{ijkl}\sigma_{ij}\sigma_{kl}$

In practice it may often be found that materials to be tested exhibit various degrees of "initial" anisotropy, that is, one or more preferred directions, due to previous history such as marked cold working, for instance. The theories of the previous sections all have initial isotropy and so cannot, strictly speaking, be applied unless the initial anisotropy is very slight. For materials showing very pronounced anisotropy initially, a theory must be developed which can take account of this. As yet little work has been done along this line largely because of the

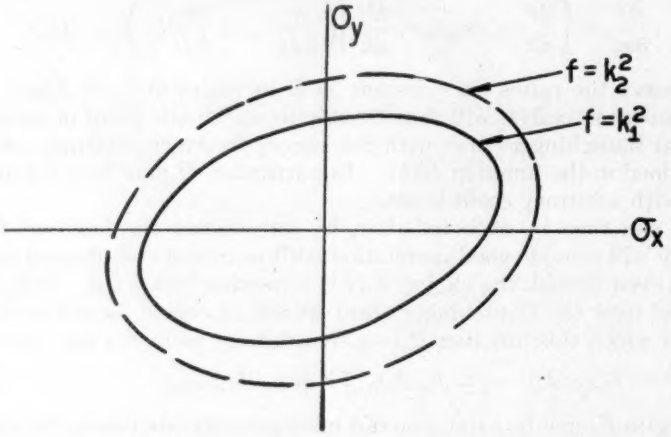

FIG. 10. Successive yield curves of the anisotropic quadratic form $f = \frac{1}{2}c_{ijkl}\sigma_{ij}\sigma_{kl}$.

complexity of all but the simplest of such theories. These theories will be dealt with only briefly here.

The simplest law of the initially anisotropic type, which is essentially an extension of the Mises criterion $f = J_2$, is the general quadratic form

$$f = \tfrac{1}{2}c_{ijkl}\sigma_{ij}\sigma_{kl}, \tag{22}$$

where c_{ijkl} (i, j, k and $l = 1, 2$ or 3) is a set of 21 independent constants since $c_{ijkl} = c_{klij}$. If the assumption of incompressibility,

$$\frac{\partial f}{\partial \sigma_{pp}} = c_{ppkl}\sigma_{kl} = 0,$$

is added, the number of independent constants is reduced to 15. The constants c_{ijkl} are, of course, determined by the amount and type of

initial anisotropy present. Since this loading function is quadratic, that is even, in the stresses equal yield stresses in tension and compression in any one direction will be predicted. However, tensile yield stresses differ from one direction to the next. In the (σ_x, σ_y) plane, Fig. 10, this loading criterion amounts simply to a rotation of the ellipse $J_2 = k^2$. Whereas the latter made four equal intercepts with the σ_1 and σ_2 axes, being inclined at 45° to them, the ellipse representing the general quadratic form makes pairwise equal intercepts with the coordinate axes, that is to say: $\sigma_x{}^t = |\sigma_x{}^c|$ and $\sigma_y{}^t = |\sigma_y{}^c|$, but $\sigma_x{}^t \neq \sigma_y{}^t$. As plastic deformation proceeds the yield curve expands uniformly about the origin as was the case in Sec. I, dealing with the corresponding isotropic law $f = J_2$. Thus this theory does not contain a Bauschinger effect. The initial anisotropy of this law is preserved throughout the deformation and so, in that sense, work hardening is again isotropic. Since $\dfrac{\partial f}{\partial \sigma_{ij}} \Big/ \dfrac{\partial f}{\partial \sigma_{kl}} = \dfrac{c_{ijpq}\sigma_{pq}}{c_{klmn}\sigma_{mn}}$ this law satisfies the assumption of constant strain ratios during radial loading.

In general $f = c_{ijkl}\sigma_{ij}\sigma_{kl}$, apart from being initially anisotropic has the same features as $f = J_2$. Thus rough correlation with test results of a single loading is found experimentally (**8–10**), whenever the initial anisotropy does not change appreciably during the experiment under consideration. This may be the case when the initial anisotropy is extremely large so that subsequent anisotropy produced by the deformation will produce little or no appreciable change.

VIII. $f = \frac{1}{2}c_{ijkl}(s_{ij} - m\epsilon_{ij}{}^p)(s_{kl} - m\epsilon_{kl}{}^p)$

If the initial anisotropy is relatively small, experiment may show that subsequent deformation produces a marked change of anisotropy. Thus the next step is to consider a theory, applicable to initially anisotropic materials which also contains a Bauschinger effect, namely

$$f = \tfrac{1}{2}c_{ijkl}(s_{ij} - m\epsilon_{ij}{}^p)(s_{kl} - m\epsilon_{kl}{}^p). \tag{23}$$

This is a direct extension of $f = J_2 - ms_{ij}\epsilon_{ij}{}^p$ treated in Sec. III. Whereas $f = J_2$ of Sec. I and $f = \frac{1}{2}c_{ijkl}\sigma_{ij}\sigma_{kl}$ of Sec. VII displayed essentially the same features (apart from initial anisotropy) this is not the case with the two corresponding strain theories above. In practice, tests on anisotropic materials (with varying, that is, small initial anisotropy) will not produce constant plastic strain ratios during radial loading. Therefore it seems reasonable that we abandon this assumption. In fact it is readily seen that this type of anisotropic theory does not lead to constant strain ratios for all but possibly a few isolated loading paths, depending on the manner in which the initial anisotropy was produced. Differentiation of f yields

$$\frac{\partial f}{\partial \sigma_{ij}} = g_{ijkl}(s_{kl} - m\epsilon_{kl}{}^p),$$

where $g_{ijkl} = c_{ijkl} - \frac{1}{3}c_{ppkl}\delta_{ij}$. The ratios

$$\frac{d\epsilon_{ij}^{p}}{d\epsilon_{kl}^{p}} = \frac{\partial f}{\partial \sigma_{ij}} \Big/ \frac{\partial f}{\partial \sigma_{kl}} = \frac{g_{ijpq}(s_{pq} - m\epsilon_{pq}^{p})}{g_{klmn}(s_{mn} - m\epsilon_{mn}^{p})}$$

will not, in general, be constant as K increases in $\sigma_{ij} = K\sigma_{ij}^{o}$, unless $c_{ijpq} = \delta_{ip}\delta_{jq}$. This reduces the theory to the isotropic form

$$f = \tfrac{1}{2}(s_{ij} - m\epsilon_{ij}^{p})(s_{ij} - m\epsilon_{ij}^{p}) = J_2 - ms_{ij}\epsilon_{ij}^{p} + m^2 I_2.$$

In Fig. 11 the solid curve is the initial yield curve, identical with that

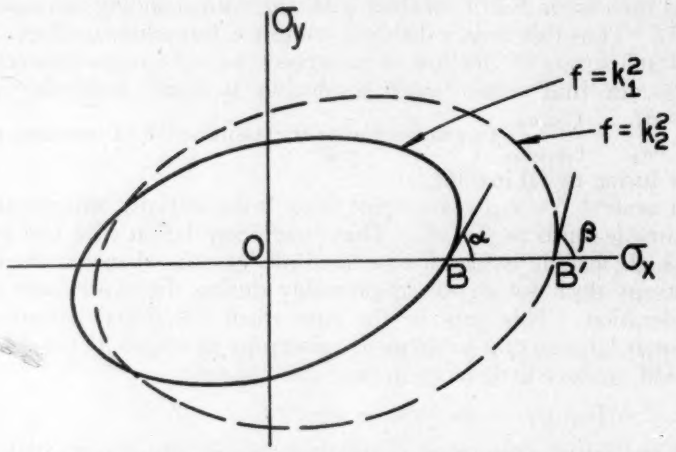

FIG. 11. Successive yield curves of $f = \frac{1}{2}c_{ijkl}(s_{ij} - m\epsilon_{ij}^{p})(s_{kl} - m\epsilon_{kl}^{p})$ in axial (σ_x) tension. α and β denote the slopes of the yield curves at successive stress points B and B'. $\alpha \neq \beta$ illustrates varying plastic strain ratios in axial tension.

in Fig. 10. The dotted curve is the yield curve after loading in axial tension (σ_x). The new curve remains an ellipse and undergoes a translation along the σ_x-axis and a uniform expansion. In addition it rotates so that tangents at successive points B and B' along a radius vector are no longer parallel. This means that strain ratios alter and that the original anisotropy is not preserved during deformation. There may exist some direction along which radial loading will produce zero rotation of the ellipse.

The usefulness of this theory to the analyst as regards correlation of experiments, will be of the same degree as that discussed in Sec. III, $f = J_2 - ms_{ij}\epsilon_{ij}^{p}$. Further generalizations of isotropic to anisotropic theories have as yet not been dealt with in theory. There seem at present to be too few experimental results to permit rational development of a useful anisotropic theory more flexible than those discussed here.

CONCLUSIONS

Eight loading criteria have been examined. Their essential features and ranges of applicability are summarized below to aid the experimenter in the choice of the most appropriate one for analysis and correlation of his data. The homogeneity restrictions listed make strain ratios constant for radial loading $\sigma_{ij} = K\sigma_{ij}^{\circ}$.

1. $f = J_2$ or $f(J_2)$ yields $\mu = \nu$ and no Bauschinger effect. It is suitable for rough correlation of simple tests, especially $\sigma_{ij} = K\sigma_{ij}^{\circ}$.

2. $f = f(J_2, J_3)$ yields any desired μ *vs.* ν curve but no Bauschinger effect. It is suitable for complete correlation of constant stress ratio tests in its general form and for rough correlation when loading paths in the (σ_1, σ_2) plane are confined to sufficiently narrow angular regions. Strain ratios are constant for $\sigma_{ij} = K\sigma_{ij}^{\circ}$ if f is homogeneous in the stresses.

3. $f = F(J_2) - ms_{ij}\epsilon_{ij}^{p}$ yields $\mu = \nu$ and a simple Bauschinger effect with one constant associated with strain and any number of constants in F. It is suitable for rough correlation of tests involving loading, unloading and re-loading, Fig. 7.

4. $f = F(J_2, J_3) - ms_{ij}\epsilon_{ij}^{p}$ yields any desired μ *vs.* ν curve and a simple Bauschinger effect. It is suitable for correlation of simple tests involving loading, unloading and re-loading and for rough correlation with tests of more complicated loading paths. Strain ratios are constant for $\sigma_{ij} = K\sigma_{ij}^{\circ}$ if, and only if, F is a function of a homogeneous function of the stresses, that is, $F = F(h)$.

5. $f = F(J_2, J_3) - H(J_2, J_3)s_{ij}\epsilon_{ij}^{p}$ yields any desired μ *vs.* ν curve and a considerably more flexible Bauschinger effect. It is suitable for good correlation of tests with complicated loading paths. Constancy of strain ratios for radial loading results if, and only if, F is of the form specified in 4 and H depends on h alone.

6. $f = F(J_2, J_3) - [p(J_2, J_3)s_{ij} + q(J_2, J_3)t_{ij}]\epsilon_{ij}^{p}$ yields a still more flexible Bauschinger effect since p is now unrestricted. Strain ratios are constant for radial loading if, and only if, F is of the form specified in 4 and q satisfies a certain simple linear differential equation.

7. $f = \frac{1}{2}c_{ijkl}\sigma_{ij}\sigma_{kl}$ contains no Bauschinger effect. Initial anisotropy is present and preserved. It is suitable for rough correlation of simple tests of materials with large initial anisotropy.

8. $f = \frac{1}{2}c_{ijkl}(s_{ij} - m\epsilon_{ij})(s_{kl} - m\epsilon_{kl})$ yields a Bauschinger effect controlled by one constant. The initial anisotropy is not preserved during deformation. This form should be useful for tests involving loading, unloading and reloading of materials with definite but relatively small initial anisotropy.

NOTATION[5]

σ_{ij} is the stress tensor whose components are $\sigma_{11} = \sigma_x$, $\sigma_{22} = \sigma_y$, $\sigma_{12} = \tau_{xy}$, etc., in the standard engineering notation.

$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$ is the stress deviator tensor, that is, $s_{kk} = s_{11} + s_{22} + s_{33} = s_x + s_y + s_z = 0$. In engineering notation $s_{11} = \frac{2}{3}[\sigma_x - \frac{1}{2}(\sigma_y + \sigma_z)]$, $s_{22} = \frac{2}{3}[\sigma_y - \frac{1}{2}(\sigma_z + \sigma_x)]$, $s_{12} = \tau_{xy}$, etc.

$\sigma_{kk} \equiv \sigma_{11} + \sigma_{22} + \sigma_{33} \equiv \sigma_x + \sigma_y + \sigma_z$ is 3 times the mean normal stress.

δ_{ij} is the Kronecker delta, that is, $\delta_{ij} = 1$ when $i = j$, $\delta_{ij} = 0$ when $i \neq j$.

ϵ_{ij}^p is the plastic strain tensor whose components are $\epsilon_{11}^p = \epsilon_x^p$, $\epsilon_{12}^p = \frac{1}{2}\gamma_{xy}^p$, etc., in the standard engineering notation. $d\epsilon_{ij}^p$ is the plastic strain increment tensor.

K denotes the monotonely increasing variable which characterizes radial loading.

k^2, k_1^2, k_2^2, k_3^2, k_4^2 denote values of the loading function f.

σ_{ij}°, s_{ij}°, ϵ_{ij}° are constant tensors appearing in $\sigma_{ij} = K\sigma_{ij}^\circ$, $s_{ij} = Ks_{ij}^\circ$, $\epsilon_{ij}^p = \alpha(K)\epsilon_{ij}^\circ$, where α is some monotonely increasing function of K.

$t_{ij} = s_{ik}s_{kj} - \frac{2}{3}J_2\delta_{ij}$ is the deviator of the square of the stress deviator s_{ij}, that is, $t_{kk} = t_{11} + t_{22} + t_{33} = 0$. In engineering notation $t_{11} = \frac{1}{9}[(\sigma_x - \sigma_y)^2 + (\sigma_z - \sigma_z)^2 - 2(\sigma_y - \sigma_z)^2 + 3\tau_{xy}^2 + 3\tau_{zz}^2 - 6\tau_{yz}^2]$, $t_{12} = \frac{1}{3}[3\tau_{zz}\tau_{yz} - \tau_{xy}(2\sigma_z - \sigma_x - \sigma_y)]$.

$J_2 = \frac{1}{2}s_{ij}s_{ij}$, $J_2^\circ = \frac{1}{2}s_{ij}^\circ s_{ij}^\circ$ are the second invariants of s_{ij} and s_{ij}°, respectively. In engineering notation $J_2 = \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]$.

$J_3 = \frac{1}{3}s_{ij}s_{jk}s_{ki}$, $J_3^\circ = \frac{1}{3}s_{ij}^\circ s_{jk}^\circ s_{ki}^\circ$ are the third invariants of s_{ij} and s_{ij}°, respectively. In engineering notation $J_3 = \frac{1}{27}(2\sigma_1 - \sigma_2 - \sigma_3)(2\sigma_2 - \sigma_3 - \sigma_1)(2\sigma_3 - \sigma_1 - \sigma_2)$.

$I_2 = \frac{1}{2}\epsilon_{ij}^p\epsilon_{ij}^p$, $I_3 = \frac{1}{3}\epsilon_{ij}^p\epsilon_{jk}^p\epsilon_{ki}^p$ are the second and third invariants of ϵ_{ij}^p. In engineering notation $I_2 = \frac{1}{6}[(\epsilon_x - \epsilon_y)^2 + (\epsilon_y - \epsilon_z)^2 + (\epsilon_z - \epsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)]$.

σ_1, σ_2, σ_3 denote the principal stresses.

s_1, s_2, s_3 denote the principal stress deviations, $s_1 = \frac{2}{3}[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)]$, etc.

$d\epsilon_1^p$, $d\epsilon_2^p$, $d\epsilon_3^p$ denote the principal plastic strain increments.

μ, ν are the Lode variables given by $\mu = \dfrac{2\sigma_2 - \sigma_3 - \sigma_1}{\sigma_1 - \sigma_3} = \dfrac{3s_2}{s_1 - s_3}$,

$$\nu = \frac{2d\epsilon_2^p - d\epsilon_3^p - d\epsilon_1^p}{d\epsilon_1^p - d\epsilon_3^p} = \frac{3d\epsilon_2^p}{d\epsilon_1^p - d\epsilon_3^p}.$$

[5] Repeated literal subscript other than x, y, and z in the same term denotes summation over that subscript from one to three, for example, $\sigma_{ij}\epsilon_{ij} = \sigma_{11}\epsilon_{11} + \sigma_{22}\epsilon_{22} + \sigma_{33}\epsilon_{33} + \sigma_{12}\epsilon_{12} + \sigma_{21}\epsilon_{21} + \sigma_{23}\epsilon_{23} + \sigma_{32}\epsilon_{32} + \sigma_{31}\epsilon_{31} + \sigma_{13}\epsilon_{13} \equiv \sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}$.

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Dry Cleaning Powder (*Wall Street Journal*, Vol. cxxxvii, No. 71).—A dry cleaning powder which the company says literally "eats and digests certain types of stains from clothing" has been perfected by the laboratories of Pabst Brewing Co. in Milwaukee, Wis.

It is an enzyme with a great affinity for protein, the company explained. Through a form of digestive action it actually "devours" stains caused by eggs, milk, ice cream, chocolate, coffee, blood, glue and other protein-base ingredients. The new spot eater will also get rid of beer stains.

The new product can be used on cotton, wool, nylon, rayon and certain acetates, according to Dr. Alexander Frieden, director of research for Pabst. It cannot be used on some fabrics manufactured from artificial yarn having a protein base since the enzyme would proceed to digest the fabric itself.

Exzyme, as the powder is known, has been tested in the Al Tash Dye Works, Grand Prairie, Texas, for three years. Milwaukee dry cleaners have been testing it more recently.

Exzyme, the company asserts, is a cheaper and safer cleaning agent than the petroleum-base products now used. Diluted in the ratio of one teaspoonful to a gallon of water, it will provide a successful solution to food spots in clothing. The powder is now in commercial production.

Radio Attachment Eliminates Commercials.—A new device which completely and automatically eliminates commercial spiels and blurbs from broadcast programs without losing so much as a note of the music was described and demonstrated at the annual convention of the Institute of Radio Engineers. It contains in its present form only four ordinary radio tubes. It can be attached to any existing set and costs little.

Invented and perfected by Dr. R. Clark Jones, mathematical physicist of the Polaroid Corporation of Cambridge, Mass., this bit of modern electronic magic "listens" for a program's pauses and measures their abruptness, remembers how many have passed during the preceding seconds of the program, and makes its decisions accordingly. It rarely fails, the inventor said, and can even distinguish between singing commercials and other music. The automatic commercial blurb-silencer was evolved by Dr. Jones from the ever-broadening field of the electronic calculator.

If manufactured and extensively applied the device is admitted to be something which might upset the economics of broadcasting. In its present form, the device resembles the inside or "works" of a midget home radio. The cost of placing it on the market was placed at something between $15 and $20.

The unit may have other uses than to comfort home listeners who are singularly sensitive or intolerant to commercials, Dr. Jones said. It may have real importance in adding to our knowledge of the nature of speech and music and the workings of the human brain.

It is completely automatic in operation when attached to a radio or television, is quite simple and reliable, but required months of research to produce. It makes use of five inherent functions of the ordinary electronic calculator—it extracts logarithms, it integrates values, differentiates, subtracts and can "remember." Dr. Jones developed the device in his spare time, as a hobby.

SOLUTION OF TRANSIENTS IN ACTIVE FOUR-TERMINAL NETWORKS

BY

HERMAN EPSTEIN [1]

SUMMARY

Matrix and operational methods together provide a compact, convenient and systematic solution of transients. This paper illustrates the application of these methods to the solution of active four-terminal networks. First, the matrices are written with the Laplacian operator, s, as the parameter. Second, the vacuum tube as a four-terminal network is reviewed. Third, an example is given which illustrates the use of the method.

I. INTRODUCTION

In a recent paper (1) [2] matrices were applied to active four-terminal networks. The ideas formulated in that paper, and previously in a series of papers (2–6), give rather completely the application of matrix algebra to obtain the steady-state response of four-terminal networks. The application of matrix algebra and operational methods at the same time offers a method to further take advantage of these ideas in the calculation of transients. This has already been done in the calculation of transients in passive, bilateral four-terminal networks (3). It is the purpose of this paper to complete the picture by developing the application of matrices and operational methods to linear, unilateral, active four-terminal networks.

The development will illustrate that the equations for transients in linear vacuum-tube circuits, including the interelectrode capacitances and with or without feedback, can be formulated easily. The main difficulty to be expected in the actual application of the method is that of finding the time function from the operational function. Extensive tables for this purpose are available (7). Otherwise, the application of the method to active networks offers the same advantages pointed out in footnote reference (3) for passive networks. These advantages are due to the compact notation and systematic procedure afforded by these methods. This results in a smaller probability of error and probably a reduction of labor in the solution of specific problems.

II. DEVELOPMENT OF THE MATRIX NOTATION

In the following, the reader is assumed to be familiar with elementary matrix algebra and operational methods. Figure 1 is a schematic dia-

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

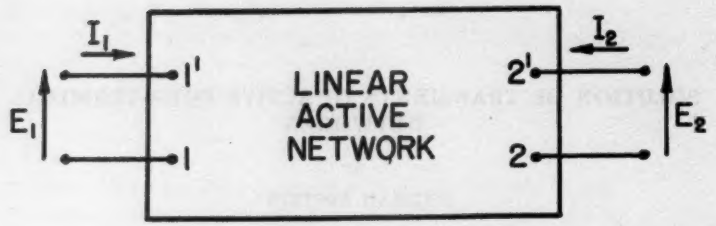


FIG. 1. Schematic diagram of a four-terminal network.

gram of an arbitrary linear active four-terminal network showing the reference directions for voltage and current. The notation is the same as that used by Guillemin (8). For the network shown, the node equations can be written with all of the functions understood to be expressed in terms of the Laplace transform operator, s. These equations are

$$\begin{aligned} I_1 &= Y_{11}E_1 + Y_{12}E_2 \\ I_2 &= Y_{21}E_1 + Y_{22}E_2. \end{aligned} \tag{1}$$

Equation 1 may be written in the convenient matrix form

$$\left\| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\| = \left\| \begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix} \right\| \cdot \left\| \begin{matrix} E_1 \\ E_2 \end{matrix} \right\|. \tag{1a}$$

If (1) is solved for E_1 and I_1, in terms of E_2 and I_2, the following equation in matrix form is obtained:

$$\left\| \begin{matrix} E_1 \\ I_1 \end{matrix} \right\| = \left\| \begin{matrix} A & B \\ C & D \end{matrix} \right\| \cdot \left\| \begin{matrix} E_2 \\ -I_2 \end{matrix} \right\|, \tag{2}$$

where

$$\begin{aligned} A &= -\frac{Y_{22}}{Y_{21}} \qquad B = -\frac{1}{Y_{21}} \\ C &= -\frac{|Y|}{Y_{21}} \qquad D = -\frac{Y_{11}}{Y_{21}} \\ |Y| &= Y_{11}Y_{22} - Y_{12}Y_{21}. \end{aligned}$$

There are two other relationships between the currents and voltages in active four-terminal networks which are usually useful. The relationships including 1 and 2 are given in Appendix I. The interrelationships between the sets of coefficients are given in Appendix II.

In vacuum-tube circuits, the coupling from grid to plate is unilateral. The mutual coefficients of the system are not equal; that is,

$$\begin{aligned} &(a) \qquad Y_{12} \neq Y_{21} \\ &(b) \qquad Z_{12} \neq Z_{21}. \end{aligned} \tag{3}$$

The ratio of the mutual coefficients may then be expressed as

$$\frac{Y_{12}}{Y_{21}} = \eta, \tag{4}$$

where η is in general a complex number. It is then also clear from the expressions given in Appendix I, that

$$(a) \qquad \frac{Z_{12}}{Z_{21}} = \eta$$

and (5)

$$(b) \qquad AD - BC = \eta.$$

In linear, bilateral systems, $\eta = 1$. In linear unilateral systems, however, η is in general a complex number. In fact, $|\eta|$ has values between zero and one and $\eta = 0$ indicates complete unilateral coupling in the grounded plate and cathode circuit.

The expressions given in Appendix I and II are those for a single four-terminal network as shown in Fig. 1. Any complicated network can usually be analyzed by judicious use of these expressions. In most cases of active four-terminal networks, however, the connection is cascade or equivalent to it, as shown in Fig. 2. The response of a

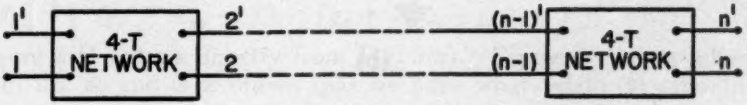


FIG. 2. Cascade connection of n four-terminal networks.

cascaded network involves the product (if the stages are different) or the power (if the stages are identical) of matrix (Eq. 21c), Appendix I. Push-pull amplifiers, for example, can be analyzed by consideration of the basic circuit of Fig. 1. In this case, the input of one tube of the push-pull pair can be considered to be received at some time after the other input. The output is merely the sum of the two transients which are displaced from each other by a fixed time.

III. EQUIVALENT FOUR-TERMINAL NETWORKS

Based upon some simplifying assumptions, the four-terminal equivalent of the three possible connections of the vacuum-tube will be developed (9). The equivalent circuit will be expressed in terms of the operational admittance matrix from which other necessary forms can be obtained using Appendices I and II.

The assumptions which are made are the following:

(a) The vacuum-tube operates in the linear range.

(b) The electron currents in the tube are continuous, single-valued functions of plate and grid voltages.

(c) The tube operates class A_1; that is, the signal never drives the grid to cut-off and the grid does not draw any electron current. There is a grid displacement current due to the interelectrode capacities.

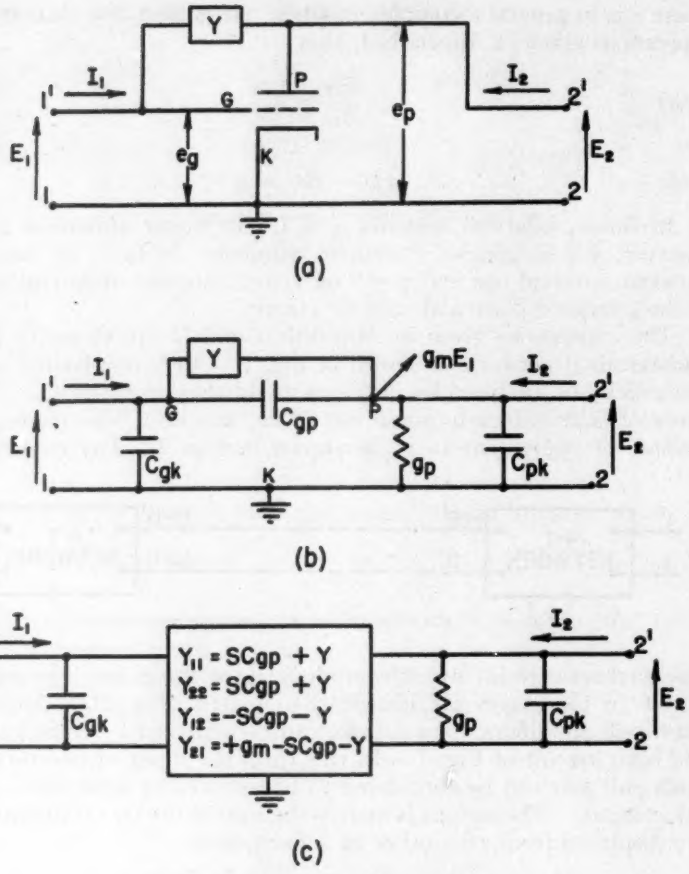


FIG. 3. Grounded cathode connection.

On the basis of these assumptions, the operation of the tube can be expressed as

$$\begin{aligned} i_g' &= 0 \\ i_p' &= g_m e_g + g_p e_p, \end{aligned} \tag{6}$$

where i_g' and i_p' are the electron currents of the grid and plate and e_g and e_p are the grid and plate signal voltages, all in terms of the operator, s. The transconductance and reciprocal plate resistance are g_m and g_p, respectively. Taking the displacement currents into account, with no additional admittance between the plate and grid, the expressions for the

total currents are

$$\begin{aligned} i_g &= s(c_{gk} + c_{gp})e_g - sc_{gp}e_p \\ i_p &= (g_m - sc_{gp})e_g + [g_p + s(c_{pk} + c_{gp})]e_p, \end{aligned} \tag{6a}$$

where g, p, k, refer to the grid, plate, and cathode, respectively.

From Fig. 3(a), the correlation between the input and output can be made. An admittance Y is placed between the plate and grid to account for any voltage feedback used with this connection. Thus, the following can be stated for Fig. 3:

$$\begin{aligned} E_1 &= e_g \qquad I_1 = i_g \\ E_2 &= e_p \qquad I_2 = i_p. \end{aligned} \tag{7}$$

Therefore, from Eq. 6, taking into account the admittance Y, we have:

$$\begin{aligned} I_1 &= [s(c_{gk} + c_{gp}) + Y]E_1 - (sc_{gp} + Y)E_2 \\ I_2 &= (g_m - sc_{gp} - Y)E_1 + [g_p + s(c_{pk} + c_{gp}) + Y]E_2. \end{aligned} \tag{8}$$

The matrix form then follows:

$$\left\| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\| = \left\| \begin{matrix} [s(c_{gk} + c_{gp}) + Y] & -(sc_{gp} + Y) \\ (g_m - sc_{gp} - Y) & [g_p + s(c_{pk} + c_{gp}) + Y] \end{matrix} \right\| \cdot \left\| \begin{matrix} E_1 \\ E_2 \end{matrix} \right\| . \tag{8a}$$

Figure 3(b) follows directly from Fig. 3(a). Figure 3(c) then follows from Eq. 8a and is so drawn that we have what Selgin (9) calls "the

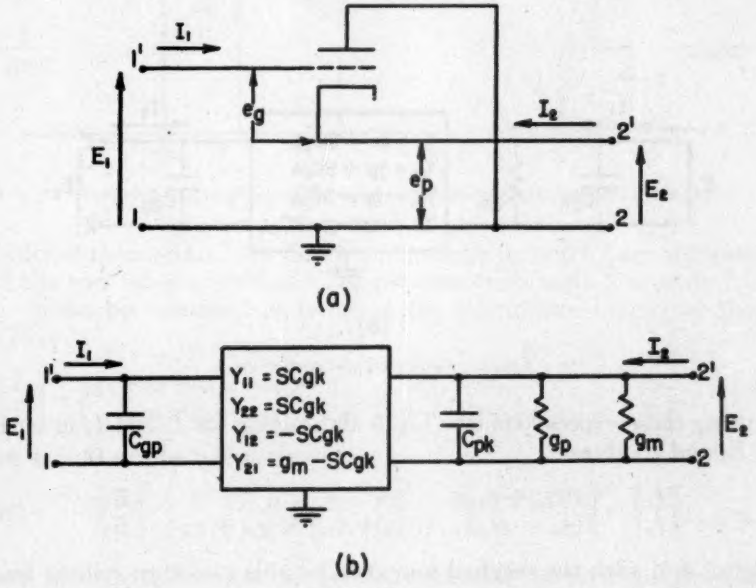
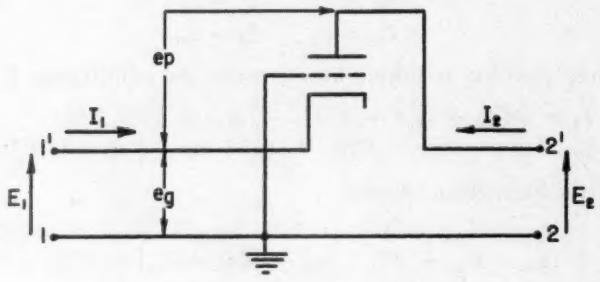


FIG. 4. Cathode-follower connection.

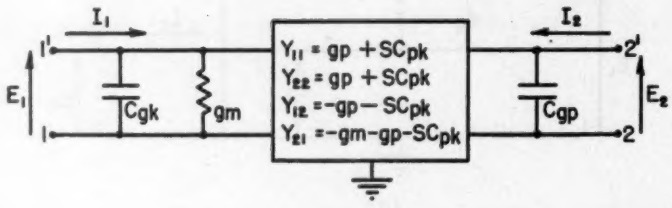
residual four-pole." Thus, the tube circuit is reduced to a four-pole and shunting admittances which can be easily lumped with the load and source admittances.

The next connection considered is the cathode-follower or grounded plate connection of Fig. 4. From Fig. 4, the following relations are evident:

$$\begin{aligned} E_1 &= e_g - e_p & I_1 &= i_g \\ E_2 &= -e_p & I_2 &= -(i_p + i_g). \end{aligned} \tag{9}$$



(a)



(b)

FIG. 5. Grounded grid connection.

Putting these expressions into Eq. 6 and solving for I_1 and I_2 in terms of E_1 and E_2 gives

$$\left\| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\| = \left\| \begin{matrix} s(c_{gk} + c_{gp}) & -sc_{gk} \\ (g_m - sc_{gk}) & s(c_{gk} + c_{pk}) + g_m + g_p \end{matrix} \right\| \cdot \left\| \begin{matrix} E_1 \\ E_2 \end{matrix} \right\|. \tag{10}$$

Figure 4(*b*) with the residual four-pole for this case then follows from the above.

Following through similarly with the grounded grid connection of Fig. 5, we can express

$$\begin{aligned} E_1 &= -e_g & I_1 &= -(i_p + i_g) \\ E_2 &= e_p - e_g & I_2 &= i_p. \end{aligned} \tag{11}$$

Solving for the operational admittance matrix,

$$\left\| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\| = \left\| \begin{matrix} g_m + g_p + s(c_{gk} + c_{pk}) & -g_p - sc_{pk} \\ -g_m - g_p - sc_{pk} & g_p + s(c_{pk} + c_{gp}) \end{matrix} \right\| \cdot \left\| \begin{matrix} E_1 \\ E_2 \end{matrix} \right\|. \tag{12}$$

IV. EXAMPLE

As an example, the response of Fig. 6 to an impulse is to be calculated. The particular circuit was chosen in order to illustrate the appli-

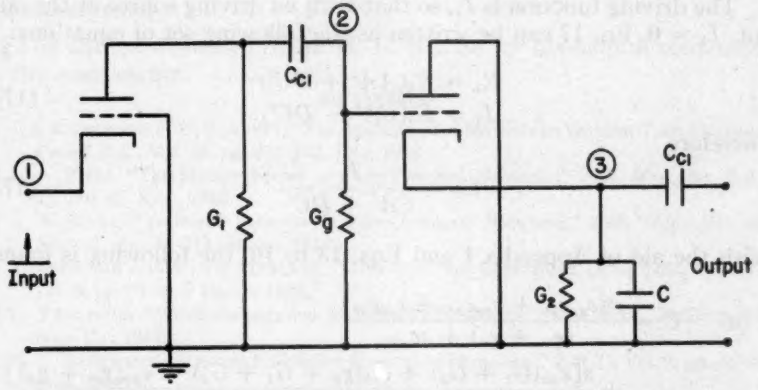


FIG. 6. Grounded-grid amplifier followed by a cathode-follower with an impulse input.

cability of the method. In the circuit, assume C_{c_1} and C_{c_2} are negligible and the two tubes are alike. The response from node 1 to node 2 in Fig. 6 can be expressed in terms of the admittance matrix so that we have

$$\left\| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\| = \left\| \begin{matrix} g_m+g_p+s(c_{gk}+c_{pk}) & -g_p-sc_{pk} \\ -g_m-g_p-sc_{pk} & g_p+G_1+G_g+s(c_{pk}+c_{gp}) \end{matrix} \right\| \cdot \left\| \begin{matrix} E_1 \\ E_2 \end{matrix} \right\|. \tag{13}$$

Now Eq. 13 can be transformed to

$$\left\| \begin{matrix} E_1 \\ I_1 \end{matrix} \right\| = \left\| \begin{matrix} A & B \\ C & D \end{matrix} \right\| \cdot \left\| \begin{matrix} E_2 \\ -I_2 \end{matrix} \right\|, \tag{14}$$

where the new coefficients can be obtained from the relations given in Appendix II.

Likewise, the response from node 2 to node 3, Fig. 6, can be expressed as

$$\left\|\begin{matrix} I_2 \\ I_3 \end{matrix}\right\| = \left\|\begin{matrix} s(c_{gk} + c_{gp}) & -sc_{gk} \\ -g_m - sc_{gk} & s(c_{gk} + c_{pk}) + g_m + g_p + G_2 \end{matrix}\right\| \cdot \left\|\begin{matrix} E_2 \\ E_3 \end{matrix}\right\| \tag{15}$$

and transformed to

$$\left\|\begin{matrix} E_2 \\ I_2 \end{matrix}\right\| = \left\|\begin{matrix} A' & B' \\ C' & D' \end{matrix}\right\| \cdot \left\|\begin{matrix} E_3 \\ -I_3 \end{matrix}\right\|. \tag{16}$$

The response of the whole circuit is then obtained by combining the matrices (14) and (16):

$$\left\|\begin{matrix} E_1 \\ I_1 \end{matrix}\right\| = \left\|\begin{matrix} AA' + BC' & AB' + BD' \\ CA' + DC' & CB' + DD' \end{matrix}\right\| \cdot \left\|\begin{matrix} E_3 \\ -I_3 \end{matrix}\right\|. \tag{17}$$

The driving function is I_1, so that with no driving source at the output, $I_3 = 0$, Eq. 17 can be written as the following set of equations:

$$\begin{aligned} E_1 &= E_3(AA' + BC') \\ I_1 &= E_3(CA' + DC'). \end{aligned} \tag{17a}$$

Therefore,

$$E_3 = \frac{I_1}{CA' + DC'}. \tag{17b}$$

With the aid of Appendix I and Eqs. 13 to 16, the following is found:

$$\begin{aligned} (a)\quad C &= \frac{s^2(c_{gp}c_{gk} + c_{gp}c_{pk} + c_{gk}c_{pk})}{(g_m + g_p) + sc_{pk}} \\ &\quad + \frac{s[c_{pk}(G_1 + G_g) + c_{gk}(g_p + G_1 + G_g) + c_{gp}(g_m + g_p)]}{(g_m + g_p) + sc_{pk}} \\ &\quad + \frac{(g_m + g_p)(G_1 + G_g)}{(g_m + g_p) + sc_{pk}}, \\ (b)\quad D &= \frac{(g_m + g_p) + s(c_{gk} + c_{pk})}{(g_m + g_p) + sc_{pk}}, \\ (c)\quad C' &= \frac{s^2[c_{gk}c_{pk} + c_{gk}c_{gp} + c_{gp}c_{pk} + c(c_{gp} + c_{gk})]}{sc_{gk} + g_m} \\ &\quad + \frac{s[g_p(c_{gk} + c_{gp}) + g_m c_{gp} + G_2(c_{gk} + c_{gp})]}{sc_{gk} + g_m}, \\ (d)\quad A' &= \frac{s(c_{gk} + c_{pk} + c) + g_m + g_p + G_2}{g_m + sc_{gk}}. \end{aligned} \tag{18}$$

Substituting the following values in (18*a–d*) and solving for (17*b*) with I_1 a unit impulse, the resulting operational equation for E_3 is

obtained:

$$\begin{array}{lll} c_{gk} = 3\ \mu\mu f & g_m = 1400\ \mu\text{mhos} & G_2 = 1\ \mu\text{mhos} \\ c_{gp} = 2\ \mu\mu f & g_p = 10\ \mu\text{mhos} & G_g = 1\ \mu\text{mhos} \\ c_{pk} = 4\ \mu\mu f & G_1 = 10\ \mu\text{mhos} & c = 100\ \mu\mu f. \end{array}$$

$$E_3 = \frac{7\times10^{-6}s^2+9830s+1.974}{10^{+6}(6464\times10^{-24}s^3+1.114\times10^{-12}s^2+9.83\times10^{-6}s+21.9)}. \quad (19)$$

The inverse Laplace transform of Eq. 19 gives the final result:

$$e_3 = [0.153\epsilon^{-4.8\times10^6 t} - 0.142\epsilon^{-4.5\times10^6 t} - 0.0087\epsilon^{-163.8\times10^6 t}]10^{12}. \quad (20)$$

The systematized approach is vividly pointed out in this example. Due to the formalized solution of the problem, the probability of errors should be decreased, with possibly less labor involved in arriving at the solution.

Acknowledgment

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APPENDIX I

$$\begin{array}{ll} (a) & \left\|\begin{matrix} I_1 \\ I_2 \end{matrix}\right\| = \left\|\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}\right\| \cdot \left\|\begin{matrix} E_1 \\ E_2 \end{matrix}\right\| \\ (b) & \left\|\begin{matrix} E_1 \\ E_2 \end{matrix}\right\| = \left\|\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}\right\| \cdot \left\|\begin{matrix} I_1 \\ I_2 \end{matrix}\right\| \\ (c) & \left\|\begin{matrix} E_1 \\ I_1 \end{matrix}\right\| = \left\|\begin{matrix} A & B \\ C & D \end{matrix}\right\| \cdot \left\|\begin{matrix} E_2 \\ -I_2 \end{matrix}\right\| \\ (d) & \left\|\begin{matrix} E_2 \\ -I_2 \end{matrix}\right\| = \left\|\begin{matrix} \alpha & \beta \\ \gamma & \delta \end{matrix}\right\| \cdot \left\|\begin{matrix} E_1 \\ I_1 \end{matrix}\right\| \cdot \end{array} \quad (21)$$

APPENDIX II

$$(a)\quad A=\frac{Z_{11}}{Z_{21}}=-\frac{Y_{22}}{Y_{21}} \qquad (i)\quad Y_{11}=\frac{D}{B}=\frac{Z_{22}}{|Z|}$$

$$(b)\quad B=\frac{|Z|}{Z_{21}}=-\frac{1}{Y_{21}} \qquad (j)\quad Y_{12}=-\frac{\eta}{B}=-\frac{\eta Z_{21}}{|Z|}$$

$$(c)\quad C=\frac{1}{Z_{21}}=-\frac{|Y|}{Y_{21}} \qquad (k)\quad Y_{21}=-\frac{1}{B}=-\frac{Z_{21}}{|Z|}$$

$$(d)\quad D=\frac{Z_{22}}{Z_{21}}=-\frac{Y_{11}}{Y_{21}} \qquad (l)\quad Y_{22}=\frac{A}{B}=\frac{Z_{11}}{|Z|}$$

$$(e)\quad Z_{11}=\frac{A}{C}=\frac{Y_{22}}{|Y|} \qquad (m)\quad \alpha=\frac{D}{\eta}=-\frac{Y_{11}}{\eta Y_{21}}$$

$$(f)\quad Z_{12}=\frac{\eta}{C}=-\frac{Y_{21}}{|Y|} \qquad (n)\quad \beta=-\frac{B}{\eta}=\frac{1}{\eta Y_{21}}$$

$$(g)\quad Z_{21}=\frac{1}{C}=-\frac{Y_{21}}{|Y|} \qquad (o)\quad \gamma=-\frac{C}{\eta}=\frac{|Y|}{\eta Y_{21}}$$

$$(h)\quad Z_{22}=\frac{D}{C}=\frac{Y_{11}}{|Y|} \qquad (p)\quad \delta=\frac{A}{\eta}-\frac{Y_{22}}{\eta Y_{21}},$$

where

$$\begin{aligned} |Y| &= Y_{11}Y_{22}-Y_{12}Y_{21} \\ |Z| &= Z_{11}Z_{22}-Z_{12}Z_{21} \\ \eta &= \frac{Y_{12}}{Y_{21}}=\frac{Z_{12}}{Z_{21}}=AD-BC. \end{aligned} \tag{22}$$

THE GOODNESS OF FIT AND THE DISTRIBUTION OF PARTICLE SIZES [1]

BY

F. KOTTLER [2]

Part II [3]

AN EXAMPLE OF THE CHI-SQUARE MINIMUM METHOD

We now illustrate the foregoing by an example: We take the emulsion, Seed Lantern No. 6102, which is represented in Figs. 1 to 4 in the present paper.

The data [4] are represented in Table I, Columns 1 to 5. Column 2 gives the end-points of the class-intervals, each of which was taken as $\Delta x = 0.2\ \mu^2$. Column 3 gives the observed class-frequencies. It will be noted that some of these numbers are not integers, as they should be, because the observer was not sure about some grains which had sizes just at the limits of two neighboring classes and, therefore, split the numbers of such grains evenly between these classes. The total number of grains is $N = 1461$. Column 4 gives the probabilities, P, which are derived from the cumulated frequencies after division by N. Column 5 gives the corresponding Normal Deviates, which are found from the equation,

$$P = \int_{-\infty}^{t} z\,dt,$$

where $z = 1/\sqrt{2\pi}\exp(-t^2/2)$. Note that the usual tables of probability functions (11) do not list this integral but the following integral,

$$\int_{-t}^{t} z\,dt = 1 - 2P \quad \text{if } t \text{ is negative or } P < 0.50$$
$$= 2P - 1 \quad \text{if } t \text{ is positive or } P > 0.50.$$

The usual arrangement of these tables is: (1), the Normal deviate, t; (2), the Normal frequency function, z; and (3), the integral, $\int_{-t}^{t} z\,dt$. Five decimal places are required for the present work uniformly for all quantities; linear interpolation is sufficient.

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[1] Communication No. 1338 from the Kodak Research Laboratories, Eastman Kodak Company, Rochester, N. Y.

[2] Kodak Park Works, Research Laboratories, Eastman Kodak Co., Rochester, N. Y.

[3] Part I appeared in this JOURNAL, May, 1951, p. 499.

[4] The data refer to Plate No. III observed by Loveland and Trivelli (2). These authors have published (*loc. cit.*) the sums of the class-frequencies obtained from eight plates. For reasons already mentioned (1), and more fully discussed below, we shall use the individual plates only.

TABLE I.—*First Approximation.*

Emulsion = Seed Lantern No. 6102, Plate III

(1)	(2)	(3)	(4)	(5)	(6) $a' = +2.32699$, $b' = +1.55529$	(7)	(8) f'	(9)	(10)
No.	x	f	P	i		P'	$N' = 1461.48229$	i'	i'^2/f'
1	0.2	608.0	0.41615	−0.21175	−0.17616	0.43008	628.55	−20.55	0.671
2	0.4	583.5	0.81553	+0.89846	+0.90189	0.81644	564.66	+18.84	0.629
3	0.6	180.5	0.93908	1.54710	1.53252	0.93730	176.63	+ 3.87	0.085
4	0.8	57.5	0.97844	2.02254	1.97994	0.97614	56.76	+ 0.74	0.010
5	1.0	21.5	0.99316	2.46555	2.32699	0.99002	20.29	+ 1.21	0.072
6	1.2	3.0	0.99521	2.59064	2.61055	0.99548	7.98	− 4.98	
7	1.4	2.0	0.99658	2.70452	2.85030	0.99782	3.42	− 1.42	
8	1.6	1.0	0.99726	2.77735	3.05798	0.99889	1.56	− 0.56	
9	1.8	3.0	0.99931	3.19880	3.24117	0.99940	0.75	+ 2.25	
10	2.0	1.0	0.99999	4.26500	3.40504	0.99967	0.39	+ 0.61	1.192
		1461.0					1460.99	+ 0.01	2.659

$P_{[3]} = 45.5\%$

The values, t, found from the tables are multiplied by 50 and plotted in millimeters with the aid of a caliper gauge, upward or downward from the mid-line (50 per cent) of the log-probability chart described in Appendix II of the previous paper (1).[5] (See Fig. 4 of the present paper.)

A graphical estimation of the best straight line fitting these data is then made. This does not contradict the statement already made (1) rejecting the use of the graphical method for size-frequency analysis. In our case, it furnishes only a first approximation, which serves as a start for the analysis and requires correction by the algebraic method.[6] Actually, the line was chosen, as already mentioned (1), such as an inexperienced observer would choose it. He would attach too much importance to the data, Numbers 5 to 9, which are situated beyond the level of 99 per cent and should, therefore, be ignored according to the rule of thumb previously given for such rough graphical estimates. (In algebraic analysis, they are weighted down by the Mueller weights if the ogive is used. Note that in our present analysis we do not use the ogive as previously (1), but the frequency polygon.) On the other hand, the inexperienced observer would attach too little importance to the point, Number 1.

The parameters, a (intercept) and b (slope), are then evaluated from the chosen line. The best way to do this is to use two abscissae, say, 0.1 and 2.4, and to determine the corresponding ordinates, t', by a caliper. One obtains so from the graph for 0.1 the value, $50t' = -62.71$ mm. and for 2.4 the value $50t' = +184.43$ mm. Substituting into the equation, $t' = a' + b' \ln x$, one obtains two linear equations, the solutions of which are

$$a' = +2.32699, \qquad b' = +1.55529,$$

by means of which the first approximation is calculated (Table I, Columns 6 to 8).

In Column 8 we give the computed class-frequencies, f', which are obtained from the computed probabilities, P', by differencing and multiplying the differences by N'. This is a corrected value obtained as follows:

$$N' = 1461/0.99967 = 1461.48229.$$

This correction is necessary to preserve the fundamental equation,

$$\Sigma f' = \Sigma f = N = 1461.$$

(The Log-Normal distribution is unlimited to the right, as will be re-

[5] The boldface numbers in parentheses refer to the references appended to this paper.

[6] If a different line were chosen for a first approximation, the final outcome after successive approximations would still be the same, within the limits of the experimental error. This proves how irrelevant the use of the graph in the present method is.

membered from the previous paper.[7] Its range, therefore, extends beyond the finite range of the observations and will contain a small additional increment of individuals in that additional range. In our case, this increment is 0.48229.)

Columns 9 and 10 contain the residuals, δ', and their evaluation for the Chi-Square test, δ'^2/f', respectively. The result of the test is $\chi^2_{[3]} = 2.650$, which corresponds to a probability of $P_{[3]} = 45.5$ per cent.

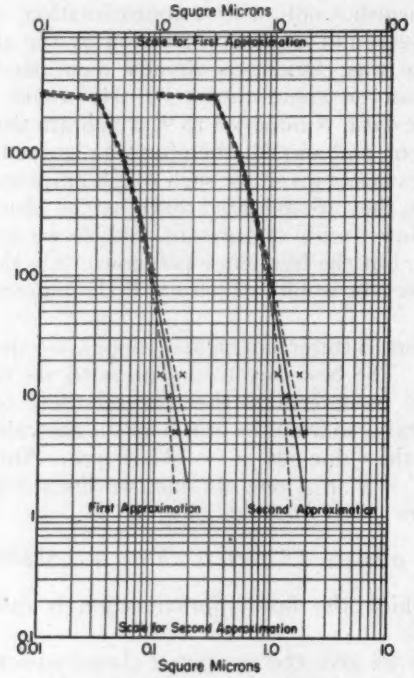

FIG. 3. Frequency polygons and error bands. (Seed Lantern No. 6102, III.)

As explained before, the small class-frequencies, Numbers 6 to 10, have been pooled for that test. The number of degrees of freedom is, consequently,

$$[6 - 2 - 1] = [3],$$

6 being the total number of the data after pooling, 2 being the number of estimated parameters, and 1 degree of freedom being lost because of the fundamental equation, (5).

[7] If we would use the limited Log-Normal distribution of that paper, which has a finite range the correction N' for N would be unnecessary. In that case, we would of course have to introduce a new parameter, x_∞, a finite quantity, for the upper limit of the distribution.

TABLE II.—*Computation of the Second Approximation.*

(1) No.	(2) s'	(3) Δ	(4) $s' \ln x$	(5) Δ	(6) s'/N'	(7) (3)+(5)+(6)	(8) $\sqrt{N'/\beta'}$	(9) A′	(10) B′	(11) E′	(12) Σ′
1	0.39280	+0.39280	−0.63219	−0.63219	−0.01407	−0.25346	1.52484	+0.59896	−0.96399	−0.02145	−0.38648
2	0.26563	−0.12717	−0.24339	+0.38880	+0.01289	+0.27452	1.60881	−0.20459	+0.62551	+0.02074	+0.44166
3	0.12329	−0.14234	−0.06298	+0.18041	+0.00265	+0.04072	2.87646	−0.40944	+0.51894	+0.00762	+0.11712
4	0.05619	−0.06710	−0.01254	+0.05044	+0.00051	−0.01615	5.07412	−0.34047	+0.25594	+0.00259	−0.08194
5	0.02661	−0.02958	0.00000	+0.01254	+0.00084	−0.01620	8.48890	−0.25108	+0.10644	+0.00713	−0.13751
6	0.01321	−0.01340	+0.00241	−0.00241	−0.00341	−0.01440	13.53330	−0.18135	+0.03262	−0.04615	−0.19488
7	0.00687	−0.00634	+0.00231	−0.00010	−0.00097	−0.00741	20.67246	−0.13106	−0.00207	−0.02005	−0.15318
8	0.00372	−0.00315	+0.00175	−0.00056	−0.00039	−0.00410	30.57089	−0.09630	−0.01712	−0.01192	−0.12534
9	0.00209	−0.00163	+0.00123	−0.00052	+0.00154	−0.00061	44.28074	−0.07218	−0.02303	+0.06819	−0.02702
10	0.00121	−0.00088	+0.00084	−0.00039	+0.00041	−0.00086	60.85806	−0.05356	−0.02373	+0.02495	−0.05234
		+0.00121√		+0.0084√	0.00000	+0.00205√		−1.14107	+0.50951	+0.03165	−0.59991√

MATRIX

$[A'A'] = +0.8146291$ $[A'B'] = -1.0327666$ $[A'E'] = -0.0169962$ $[A'\Sigma'] = -0.2351337$

$[B'B'] = +1.6691279$ $[B'E'] = +0.0356045$ $[B'\Sigma'] = +0.6719658$

$[E'E'] = +0.0089521$

The probability just found indicates that the application of the Log-Normal distribution to the data is justified because it is well above the rejection level of 10 per cent. However, the fit can be much improved by a second approximation so that the probability will increase considerably.

Before going into the second approximation, we turn to a graphical representation of the first approximation by means of the frequency polygon and its accompanying error band (see Fig. 3, left-hand side). The frequency polygon is indicated by a solid line, and the error band is indicated by broken lines at both sides of the solid line. (The error band is defined as

$$f' \pm \sigma,$$

where σ is given by Eq. 7.)

The goodness of fit of this first approximation can now be judged approximately by the position of the data with respect to the error band. There are ten data. We need not take into account the small class-frequencies, Numbers 6 to 10. Of the larger class-frequencies, the first two do not fit very well, because they are just at the borders of the error band. The next three fit well, because they are near the mid-line of the error band. Therefore, the fit can be improved if the first two can be made to fit the frequency polygon better by a second approximation. This will actually be the case.

Turning now to the second approximation, we undertake it along the lines explained before, and shown in Table II, Columns 1 to 12. We have:

$$f'(x, a', b') = \Delta \left\{ N' \int_{-\infty}^{a'+b' \ln x} z \, dt \right\}$$

$$\frac{\partial f'}{\partial a'} = \Delta\{N'z'\}$$

$$\frac{\partial f'}{\partial b'} = \Delta\{N'z' \ln x\},$$

where the operator, Δ, signifies differencing between two consecutive values. (The cumulative frequency is: $F' = N' \int z \, dt$; consequently, the class frequency is: $f' = \Delta F'$.) We also have: $\delta' = f - f'$. The weight is: $w = 1/f'$ according to Eq. 7. From these formulae, we can compute the Normal matrix as indicated in Eqs. 1*b*.

For practical reasons, however, we shall divide these values by N', thereby reducing them to the order of magnitude of unity (instead of the order of 1000). This is preferable, especially if one wants to keep a constant number of decimals, for example, five, throughout the calculation. Otherwise, one would have to change the number of decimals continuously in forming the products of the Normal matrix, according to the order of the respective multipliers. We may note here the so-

TABLE III.

Normal Equations

$$+ 0.8146291\alpha' - 1.0327666\beta' = - 0.0169962 \qquad - 0.2351337$$
$$- 1.0327666\alpha' + 1.6691279\beta' = + 0.0356045 \qquad + 0.6719658$$

$$\alpha' = + 0.0286656 \qquad \beta' = + 0.0390679$$

Computation of the Standard Errors

$$7\epsilon^2 = + 0.0089521 - (+ 0.0286656 \times - 0.0169962) - (+ 0.0390679 \times + 0.0356045) = + 0.0080483$$
$$\epsilon = 0.03390$$

$$\frac{\alpha_{11}}{\sqrt{\alpha_{11}}} = 5.6944802 \qquad \frac{\alpha_{22}}{\sqrt{\alpha_{22}}} = 2.7792292$$
$$\sqrt{\alpha_{11}} = 2.38631 \qquad \sqrt{\alpha_{22}} = 1.66710$$

$$s_{\alpha'} = \pm 0.03390 \times 2.38631 \qquad s_{\beta'} = \pm 0.03390 \times 1.66710$$

Results

$$a'' = a' + \alpha' = 2.35566 \pm 0.08092 \qquad b'' = b' + \beta' = 1.59436 \pm 0.05653$$

called *rounding error* which is unavoidable in dealing with decimal numbers. It often outweighs the *experimental error*. There is no way to escape it except by taking more decimal digits than are warranted by the experimental error, during the computation, and dropping the excessive decimals at the end of the computation. For example, we shall take seven decimals for the Normal equations and return to five decimals only after they have been solved.

The values obtained after division by N' are listed in Table II, Columns 3, 5, and 6. Column 7, which is the sum of Columns 3, 5, and 6, has been added for the application of the *summary check* (see Whittaker and Robinson (3, p. 211).

These values have now to be weighted according to the Pearson weights, $w = 1/f'$. As explained before, it is customary to use the square roots of the weights, shown in Column 8, as $\sqrt{N'/f'}$. The factor, N', has been added under the square root because of the afore-mentioned reduction to the order of unity.

The weighted values, A′, B′, and E′, which refer to the increments, α', β', and the errors, δ', respectively, are shown in Columns 9 to 11. Column 12, which is the sum of Columns 9, 10, and 11, has again been added for the application of the summary check. Note that Column 12 is first computed by weighting Column 7 and then comparing with the actual sum of Columns 9 to 11. If the latter sum deviates by one or two units of the last digit, the sum in Column 12 must be corrected, which is indicated by underscoring the last digit. A larger correction is indicative of an error in the computation. The total sums of each of Columns 9, 10, and 11 are also indicated. Their sum must equal the total sum of Column 12.

The Normal matrix is now formed in seven decimals (as just explained). An extra (fourth) column is carried for the application of the summary check. This check is very important, since the majority of the mistakes of calculation occur in the formation of the matrix.

Finally, the Normal equations are formed and solved by the Method of Successive Eliminations (see Table III). The solutions are:

$$\alpha' = +0.02867$$
$$\beta' = +0.03907.$$

In order to judge the finality of this approximation, we must calculate the standard errors of the solutions. From the formulae, (2) and (3), we find that

$$s_{\alpha'} = \pm 0.08092$$
$$s_{\beta'} = \pm 0.05653.$$

Hence, the solutions are within the limits of the standard errors. Therefore, the second approximation can be regarded as final and no third approximation is necessary.

TABLE IV.

	Second Approximation					Frequency Polygon and Error Band			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
No.	$a'' = +2.35566$ $b'' = +1.59436$	P''	$N'' = 1461.39458$	δ''	δ''^2/f''	x	$f''/\Delta x$	$\frac{(f''+\sqrt{f''})}{\Delta x}$	$\frac{(f''-\sqrt{f''})}{\Delta x}$
1	−0.21037	0.41669	608.95	−0.95	0.001	0.1	3044.75	3168.10	2921.40
2	+0.89476	0.81454	581.42	+2.08	0.007	0.3	2907.10	3027.65	2786.55
3	1.54123	0.93837	180.96	−0.46	0.001	0.5	904.80	972.50	837.55
4	1.99989	0.97724	56.80	+0.70	0.009	0.7	284.00	321.70	246.30
5	2.35566	0.99076	19.76	+1.74	0.153	0.9	98.80	121.50	76.55
6	2.64634	0.99593	7.56	−4.56		1.1	37.80	51.55	24.50
7	2.89211	0.99809	3.16	−1.16		1.3	15.80	24.70	6.90
8	3.10501	0.99905	1.40	−0.40		1.5	7.00	12.90	1 10
9	3.29281	0.99950	0.66	+2.34		1.7	3.30	7.35	
10	3.46079	0.99973	0.34	+0.66	0.742	1.9	1.70	4.60	
			1461.01	−0.01	0.913				

$P_{[3]} = 82.2\%$

Some Statistical Constants

Median = $\exp(-2.35566/1.59436) = 0.22821$

Mode = $0.22821 \times \exp[-(1/1.59436)^2] = 0.15399$

Mean = $0.22821 \times \exp[+\frac{1}{2}(1/1.59436)^2] = 0.27782$

Coefficient of variation = $\sqrt{\exp(1/1.59436)^2 - 1} = 0.69426$

The corrected values of the parameters are, now,

$$a'' = a' + \alpha' = 2.32699 + 0.02867 = 2.35566$$
$$b'' = b' + \beta' = 1.55529 + 0.03907 = 1.59436.$$

To these values may, of course, be added the standard errors. Hence, the intercept, a'', may be any value between the limits, $2.35566 - 0.08092 = 2.27474$ and $2.35566 + 0.08092 = 2.43658$, so far as the experimental accuracy goes. We shall compute the new class-frequencies, f', with the mean values of these limits.

This is done in Table IV, Columns 2 to 4; in Columns 5 and 6 the new Chi-Square is computed, which is

$$\chi^2_{[3]} = 0.913.$$

This is much lower than the corresponding value for the first approximation, which was 2.659. Consequently, the probability is now

$$P_{[3]} = 82.2\%,$$

which is nearly twice as much as the corresponding value for the first approximation, which was 45.5 per cent. This shows that the graphical estimate, made by the inexperienced observer, with which we started as a basis of the algebraic operations, was a poor one. It has now been corrected with the help of a second approximation. The result would be the same, within the limits of the experimental error, if we had started with a different, better, first approximation. Also, the results obtained represent the best possible fit, if judged by the Chi-Square test.

The corresponding results are shown graphically in Fig. 3, right-hand side. Comparing the first five data with the corresponding points of the first approximation at the left-hand side of Fig. 3, we see that all five are well within the error band and so represent a better fit to the new frequency polygon.

In Fig. 4 (see Fig. 6 in the previous paper (1)) are shown the ogives corresponding to both approximations, the first being indicated by a broken line, and the second by a solid line. The poor choice of the first approximation has already been discussed (1). We want to point out here that the error bands shown in the previous paper were computed with *Urban weights*, that is, from

$$F' \pm \sqrt{F'(1 - F'/N)},$$

where F' is the computed *cumulative* frequency, as is customary in Probit Analysis. In the present paper, we use *Pearson weights*, as should be done in size-frequency analysis. Hence, Fig. 3 gives the class-frequencies, f', and the error band computed from

$$f' \pm \sqrt{f'}.$$

Table IV contains, moreover, the values of some statistical constants (median, mode, mean, and coefficient of variation), while Table V compares the frequency curve and the frequency polygon. These data have been used in constructing Figs. 1 and 2.

HETEROGENEITY

We come now to an important subject which has been mentioned briefly in the previous paper (1).

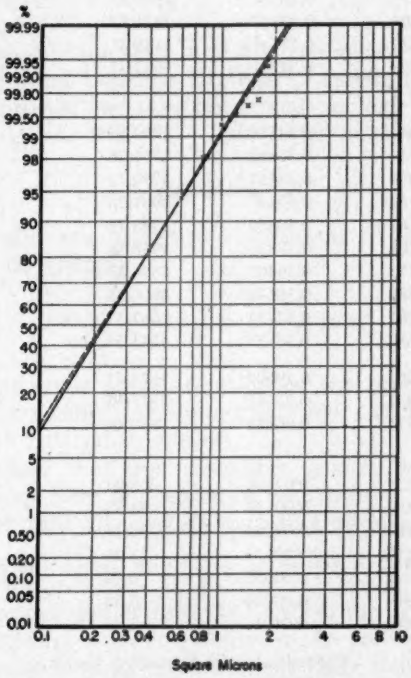

FIG. 4. Ogive, *first* and *second* approximations. (Seed Lantern No. 6102, III.)

Many authors in size-frequency analysis have drawn (for technical reasons) sub-samples from a given sample and have averaged the observations over these sub-samples. These averages were then subjected to graphical and numerical analysis, although these sub-samples were each large enough for separate analysis. This was done in the belief that it would increase the goodness of fit.

For example, see Martin *et al.* (12), and Austin (13), particularly the latter's remarks on page 335, *loc. cit.*: ". . . The most severe test is . . . made with data on ground quartz reported by Martin. . . . These

TABLE V.

(1)	(2)	Frequency Curve (3)	(4)	Frequency Polygon (5)
x	v''	z''	$N''\frac{\partial F''}{\partial x}$ $N'' = 1461.39458$	$\frac{N''\Delta F''}{\Delta x}$
0.01	−4.35943	0.00003	0.37	
0.02	−3.25430	0.00200	24.86	
0.03	−2.60785	0.01331	165.42	
0.04	−2.14918	0.03438	427.28	
0.05	−1.79340	0.07989	992.88	
0.06	−1.50272	0.12899	1603.10	
0.07	−1.25695	0.18106	2250.23	
0.08	−1.04405	0.23132	2874.87	
0.09	−0.85627	0.27650	3436.37	
0.10	−0.68829	0.31480	3912.37	3044.75 (Fig. 1, points P', P)
0.11	−0.53631	0.34550	4293.91	
0.12	−0.39759	0.36862	4581.25	
0.13	−0.26998	0.38467	4780.72	
0.14	−0.15182	0.39437	4901.28	
0.15	−0.04182	0.39859	4953.72	
0.16	+0.06107	0.39820	4948.88	
0.17	0.15773	0.39401	4896.80	
0.18	0.24886	0.38678	4806.95	
0.19	0.33507	0.37717	4687.51	
0.20	0.41684	0.36575	4545.58	
0.25	0.77262	0.29600	3678.72	
0.30	1.06331	0.22671	2817.58	2907.10
0.50	1.87774	0.06843	850.46	904.80 (Fig. 1, points Q', Q)
0.70	2.41421	0.02164	268.94	284.00
0.90	2.81489	0.00759	94.33	98.80
1.10	3.13482	0.00293	36.41	37.80
1.30	3.40117	0.00123	15.29	15.80
1.50	3.62932	0.00055	6.84	7.00
1.70	3.82889	0.00026	3.23	3.30
1.90	4.00621	0.00013	1.62	1.70

Formulae for the Frequency Curve
(Ref. (1), Appendix I, Eq. 9)

$$N''\frac{\partial F''}{\partial x} = N''b'' \times \exp(+a''/b'' + 1/2b''^2)z''$$

$$= 2329.98906 \times \exp(1.47749 + 0.19670)z'' = 12428.11505z''$$

$$z'' = \exp\left(-\frac{v''^2}{2}\right)\Big/\sqrt{2\pi}$$

$$v'' = z'' + 1/b'' + b'' \ln x = 2.98287 + 1.59436 \ln x$$

measurements include observations over the whole range of sizes and final values are the average of nine gradings of a single powder." Likewise, Sheppard *et al.* (14) use averages of four sets of observations drawn from the same emulsion. See their paper, *loc. cit.*, Tables IV and VII.

The same procedure has been used by Loveland and Trivelli (2), p. 377, who use the means of three separate determinations.

This procedure is incorrect because it disregards any heterogeneity which may exist between the sub-samples. To see that such heterogeneity exists in most cases, let us go back to the law of exponential growth which has been discussed previously (1). According to this law, the growth of the sizes is governed by the differential equation,

$$\frac{dx}{dt} = kx,$$

where x is the size, t is the time, and k is the velocity constant of the growth. Bear in mind that the latter is an essential constant of the physicochemical process and does not change as long as the conditions of this process do not change. This differential equation is integrated by

$$x = \exp k(t - t_0),$$

where t_0 is an arbitrary integration constant. Taking natural logarithms, this can be written

$$t = t_0 + \frac{1}{k} \ln x = a + b \ln x,$$

which is the logarithmic transformation of the preceding paper (1). It transforms the normally distributed variable, t (time), into the log-normally distributed variable, x (size).

It is now obvious that the parameter, $a = t_0$, will be subject to irregular fluctuations from one region of the emulsion to another, since it is quite unlikely that all crystals start to grow simultaneously from their nuclei.[8] On the other hand, the parameter, $b = \frac{1}{k}$, will be constant for the same emulsion, within the limits of the experimental error. We must therefore expect heterogeneity in the parameter, a, between different parts of the same sample, whereas there will be homogeneity in the parameter, b. Homogeneity in a is, of course, not impossible, only improbable. The only way to find out whether it exists is to analyze the divers sub-samples separately instead of analyzing only their averages, as the afore-mentioned authors have done.

Figure 5 shows what could happen if two distributions, which are heterogeneous in one parameter, are pooled together so as to average them. This figure shows two distributions which have the same variance but different means. By pooling, there results a distribution which

[8] This is why one could not reasonably be interested in an average value of the parameter, $a = t_0$. Otherwise, the averaging of the sub-samples could be justified in order to obtain such an average value.

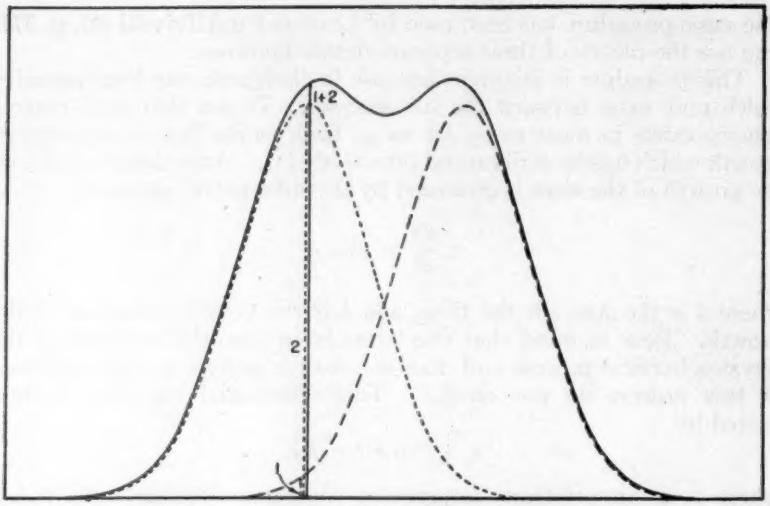

FIG. 5. Pooling of two heterogeneous distributions.

is quite dissimilar to the two original distributions, being bimodal instead of unimodal. Instead of gaining goodness of fit, one loses therefore by such a procedure.

We will now show that the eight sub-samples (plates) used by Loveland and Trivelli (2) were heterogeneous in the parameter, a, and homogeneous in the parameter b.

Let us look at the results of the estimation of the parameters in the divers cases. In Table VI we represent the values of the parameters of the eight plates from the emulsion, Seed Lantern No. 6102, computed from the measurements of Loveland and Trivelli (2). We include also the standard errors, computed from the formulae (2) and (3) and the sizes, N, of the sub-samples which show that these are sufficiently large so that they could be analyzed separately.

TABLE VI.—*Parameters of Seed Lantern No. 6102.*

Plate (sub-sample)	a''	b''	N (size of sub-sample)
I	2.033 ± 0.068	1.663 ± 0.052	1964
II	2.213 ± 0.057	1.572 ± 0.041	1994
III	2.356 ± 0.081	1.595 ± 0.057	1461
IV	2.116 ± 0.101	1.541 ± 0.073	1425
V	2.389 ± 0.086	1.666 ± 0.061	1553
VI	2.089 ± 0.063	1.515 ± 0.046	1702
VII	2.107 ± 0.056	1.512 ± 0.040	1883
VIII	1.985 ± 0.064	1.539 ± 0.047	1552

In Fig. 6 there is given a chart of the standard error bands for the eight parameters, a, and for the eight parameters, b. It can be seen at a glance that the values, b, show much less scatter than the values, a. This is borne out by a closer examination of Fig. 6. We have drawn the limits of the error bands, omitting the mid-lines which represent the computed values of the parameter. Since we do not know the true values, any value which falls within the limits of the band is a possible value and equal to the computed value within the limits of the experimental error. Hence, the maximum distance between two such values must not exceed twice the width of the standard error. In our case, there are eight different standard errors. We choose the largest of

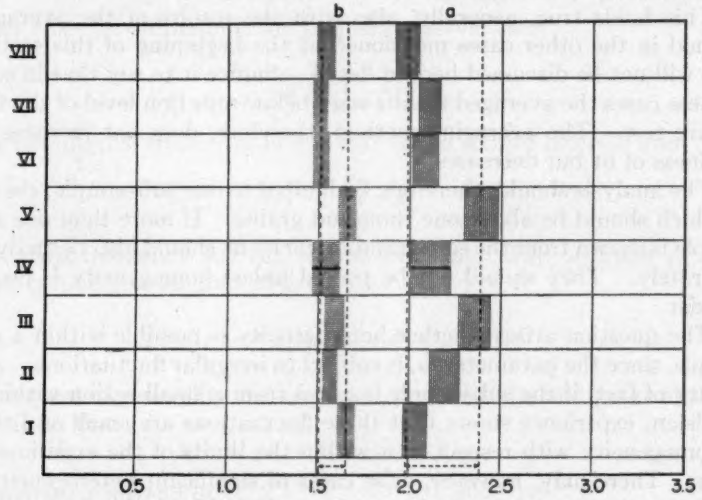

FIG. 6. Error bands for the parameters, a, b, of the eight emulsions. (Seed Lantern No. 6102.)

them to make sure that we take into account the maximum tolerance. The maximum standard error shown in Fig. 6 is that of Plate IV and is indicated by an arrow (solid line). Now we note the extreme mid-lines to the left and to the right which represent the two computed values which have the largest distance from each other. This distance is indicated by an arrow (broken line) at the bottom of the figure. It will be seen that the two arrows are just equal. Hence, all the eight values of the parameter, b, derived from computation, are equal within the maximum limits of the experimental error, as defined above. We may, therefore, state that there is *homogeneity* with respect to the parameter, b, among the eight plates chosen from the one sample, No. 6102, of the emulsion, Seed Lantern.

It is quite a different picture, however, if we turn now to the eight error bands drawn in Fig. 6 for the eight computed values of the parameter, a. Repeating the procedure sketched above, we obtain a solid arrow again with Plate IV, which we shall use as a measure of tolerance for the experimental error. Drawing the extreme mid-lines to the left and to the right, we find now a broken arrow which is twice the length of the solid arrow. Therefore, there is definitely heterogeneity with respect to the parameter, a, among the eight plates.

This shows that there should be no averaging of the eight plates. In any case, if the averaging is done, the Chi-Square test applied to its result shows a worse probability than the eight individual probabilities corresponding to the eight plates.

This holds true, generally, also with the results of the averaging method in the other cases mentioned at the beginning of this section. This will not be discussed here in detail. Suffice it to say that in some of these cases the averaged results were below rejection level of the Chi-Square test. The averaging method, therefore, does not increase the goodness of fit but decreases it.

The analysis should, therefore, be limited to one sub-sample, the size of which should be about one thousand grains. If more than one sub-sample is drawn from the same sample, each one should also be analyzed separately. They should not be pooled unless homogeneity is proved to exist.

The question arises whether heterogeneity is possible within a sub-sample, since the parameter, a, is subject to irregular fluctuations. As a matter of fact, if the sub-sample is taken from a small region within an emulsion, experience shows that these fluctuations are small and there is homogeneity with respect to a within the limits of the experimental error. There may, however, arise cases of significant heterogeneity in the parameter, a, and also in the parameter b, within a sub-sample. How to deal with such cases will be the subject of a later paper.

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APPENDIX I

Frequency Polygon and Frequency Curve

Let z be the ordinate of the frequency curve, then

$$F_2 = \int_{-\infty}^{t_2} z\,dt$$

is the cumulative frequency at the end of the second interval and

$$f_2 = \int_{t_1}^{t_2} z\,dt$$

is the class-frequency for the second interval. The ordinate of the frequency polygon for that interval is

$$f_2/(t_2 - t_1) = \int_{t_1}^{t_2} z\,dt/(t_2 - t_1).$$

Without loss of generality, we may take $t = 0$ for the mid-point of that interval, so that

$$t_2 = -t_1 = (\text{say})\ \theta$$

and, writing f instead of f_2,

$$f/2\theta = \int_{-\theta}^{+\theta} z\,dt/2\theta.$$

Developing z into a Taylor series around $t = 0$, we have

$$f/2\theta = \int_{-\theta}^{+\theta} (z_0 + tz_0' + t^2z_0''/2 + \cdots)dt/2\theta$$
$$= z_0 + \theta^2z_0''/6 + \theta^4z_0''''/120 + \cdots,$$

where the primes denote differentiation with respect to t. The odd powers of θ cancel in the result.

Therefore, we have, for the difference between frequency polygon and frequency curve, the following approximate expression:

$$f/2\theta - z_0 = \theta^2z_0''/6 + \cdots.$$

That difference is *positive* or *negative* according to the sign of the second derivative of the ordinate, z, at the mid-point of the interval. It becomes zero only at the inflection point where $z'' = 0$. Hence, if the slope, z', decreases, the ordinate of the frequency polygon is *below* the corresponding ordinate of the frequency curve. This is the case at the mid-point, 0.1, of the first interval so that P is very much below P'. (See Fig. 1.) The curvature of the frequency curve, z, is at that point large; therefore, the slope decreases rapidly within the first interval. If, on the other hand, the slope increases again, which is the case beyond the inflection point, for instance, at 0.5, the mid-point of Class No. 3, the ordinate of the frequency polygon, Q, is *above* the corresponding ordinate of the frequency curve, Q'. That difference is, however, not as large as in the first case, since the slope increases there slowly.

Note also the influence of the width of the interval. We have

$$2\theta = \Delta \ln x = \Delta x/x$$

for that width, and it enters in the formula for the difference between frequency polygon and frequency curve with the second power. Therefore, that difference is large when x is small. This is a further reason to render that difference as large as it is at the mid-point of Class No. 1.

This difference at that special mid-point between the datum which is represented by the frequency polygon and the computed value which is usually represented by the frequency curve, has been sometimes misinterpreted. It has been ascribed to physical rather than to mathematical reasons and has been explained by the failure of the microscope to detect some particles because of its limit of resolving power. Consequently, an extrapolation has been used to make up for that failure by bringing the datum up to the frequency curve. This procedure cannot be accepted. As we have seen, the reason for the difference between the datum and the frequency curve is the same as the reason for the difference between the frequency polygon and the frequency curve. The datum always corresponds to the former and not to the latter. That difference is, therefore, to be expected. This is confirmed by the cases where the classification is such that the first mid-point is beyond the inflection point. Then the difference changes sign, and one would have to assume that a surplus of particles instead of a deficit is furnished by the experimental evidence, for example, that it includes some particles which are not silver halides but impurities.

We shall not go into the controversial question of whether or not optical reasons may account for the occurrence of such a surplus or deficit of particles. However, the extrapolation mentioned above is strictly forbidden by the rules of the calculus of observations. It means that a fundamental datum of the experimental evidence, *viz.*, the total number of grains, N, is arbitrarily changed. *No datum should be changed or omitted without very stringent reasons.* If such reasons exist, they can,

however, become apparent only after the performance of the algebraic operations and not before. Especially a changed number, N, should not be used for constructing or computing the ogive, because such a change also alters the geometric nature of that line as need not be pointed out more extensively.

As a matter of fact, in all our computations which are reported in what follows, we have not found a single case with a large residual in Class No. 1 as would have to be the case if there had been either too few or too many particles in that class. Statistically speaking, there is, therefore, no reason to assume a significant failure of the microscope in either direction.

We repeat again as in the text that we do not use here the frequency curve. Only the frequency polygon is used in this paper.

APPENDIX II

The Standard Errors of the Parameters

Let the observations be

$$y_i \qquad i = 1, 2, 3, \cdots n.$$

They are subject to error and fluctuate if repeated, around some mean values, $\overline{y_i}$, which, for simplicity, we shall take all equal to zero:

$$\overline{y_i} = 0 \qquad i = 1, 2, 3, \cdots n.$$

They are assumed to be independent of each other; therefore,

$$\text{covar } (y_i, y_k) = \overline{y_i y_k} = 0 \qquad i, k = 1, 2, 3, \cdots n \ (i \neq k)$$

and, also for simplicity, to carry each the same weight, therefore,

$$\text{var } (y_1) = \text{var. } (y_2) = \cdots = \text{var. } (y_n) = (\text{say}) \ \epsilon^2$$

or

$$\overline{y_1^2} = \overline{y_2^2} = \cdots = \overline{y_n^2} = \epsilon^2.$$

Let these observations be represented as a linear regression on the independent variables, x_i, $i = 1, 2, \cdots n$, which are assumed to be free from error. Hence,

$$y_i = a + bx_i \qquad i = 1, 2, 3, \cdots n.$$

We apply the method of least squares and obtain solutions, a', b', for the two parameters and residuals, δ_i', such that

$$y_i = a' + b'x_i + \delta_i'.$$

We have the Normal equations:

$$\sum y_i = a'n + b' \sum x_i$$
$$\sum y_i x_i = a' \sum x_i + b' \sum x_i^2.$$

The Normal matrix is therefore:

$$a_{11} = n \qquad a_{12} = \sum x_i \qquad a_{13} = \sum y_i$$
$$a_{21} = \sum x_i \qquad a_{22} = \sum x_i^2 \qquad a_{23} = \sum y_i x_i.$$

The inverse matrix to the Normal determinant is:

$$\alpha_{11} = \sum x_i^2/A \qquad \alpha_{12} = -\sum x_i/A$$
$$\alpha_{21} = -\sum x_i/A \qquad \alpha_{22} = n/A,$$

where $A = n\sum x_i^2 - (\sum x_i)^2$ is the Normal determinant.

The solutions are:

$$a' = \alpha_{11}\sum y_i + \alpha_{21}\sum y_i x_i$$
$$b' = \alpha_{12}\sum y_i + \alpha_{22}\sum y_i x_i,$$

and so are linear functions of the n observations:

$$a' = \sum A_i y_i, \qquad \text{where} \qquad A_i = \alpha_{11} + \alpha_{21}x_i,$$
$$b' = \sum B_i y_i, \qquad \text{where} \qquad B_i = \alpha_{12} + \alpha_{22}x_i.$$

Note that:

$$\sum A_i = 1 \qquad \sum A_i x_i = 0$$
$$\sum B_i = 0 \qquad \sum B_i x_i = 1.$$

Also,

$$\begin{aligned}\sum A_i^2 &= \sum (\alpha_{11} + \alpha_{21}x_i)^2 \\ &= \alpha_{11}(\alpha_{11}n + \alpha_{21}\textstyle\sum x_i) + \alpha_{21}(\alpha_{11}\sum x_i + \alpha_{21}\sum x_i^2) \\ &= \alpha_{11}(\alpha_{11}a_{11} + \alpha_{21}a_{21}) + \alpha_{21}(\alpha_{11}a_{12} + \alpha_{21}a_{22}) \\ &= \alpha_{11},\end{aligned}$$

where we have used the properties of the inverse matrix. Similarly,

$$\sum A_i B_i = \alpha_{12} = \alpha_{21}$$
$$\sum B_i^2 = \alpha_{22}.$$

It is now easy to find the variances of the parameters; their square roots are the desired standard errors. By a well-known rule (Kendall (8), Vol. I, p. 208), we have:

$$\begin{aligned}\text{var}\,(a') = {} & A_1^2\,\text{var}\,(y_1) + A_2^2\,\text{var}\,(y_2) + \cdots + A_n^2\,\text{var}\,(y_n) \\ & + 2A_1A_2\,\text{covar}\,(y_1, y_2) + \cdots + 2A_{n-1}A_n\,\text{covar}\,(y_{n-1}, y_n).\end{aligned}$$

Substituting, we obtain:

$$\text{var}\,(a') = A_1^2\epsilon^2 + A_2^2\epsilon^2 + \cdots + A_n^2\epsilon^2 = \alpha_{11}\epsilon^2.$$

Similarly,

$$\text{var}\,(b') = B_1^2\epsilon^2 + B_2^2\epsilon^2 + \cdots + B_n^2\epsilon^2 = \alpha_{22}\epsilon^2,$$
$$\text{covar}\,(a', b') = A_1B_1\epsilon^2 + A_2B_2\epsilon^2 + \cdots + A_nB_n\epsilon^2 = \alpha_{12}\epsilon^2.$$

These formulae were first derived in a similar way by Gauss (15), in 1823. They are equivalent to the formulae, (2), of the text.

The Chi-Square is defined, in our case,

$$\chi^2 = \sum \delta_i'^2/\epsilon^2,$$

since the standard errors of all the observations are equal. It can be written:

$$\chi^2 = \sum (y_i - a' - b'x_i)^2/\epsilon^2$$
$$= \{\sum y_i^2 - a' \sum y_i - b' \sum y_ix_i + a'[a'n + b' \sum x_i - \sum y_i] + b'[a' \sum x_i + b' \sum x_i^2 - \sum y_ix_i]\}/\epsilon^2,$$

which, by the Normal equations (8), reduces to

$$\chi^2 = \{\sum y_i^2 - a' \sum y_i - b' \sum y_ix_i\}/\epsilon^2. \tag{9}$$

This is also a formula due to Gauss. It is used for an indirect calculation of the Chi-Square instead of its direct calculation from the residuals, δ'. (See, however, below.)

Finally, we can find the mean value and the variance of the Chi-Square. We have, from (9):

$$\overline{\chi^2} = \{\overline{\sum y_i^2} - \overline{\sum A_i y_i \sum y_k} - \overline{\sum B_i y_i \sum y_k x_k}\}/\epsilon^2$$
$$= \{n\epsilon^2 - \epsilon^2 \sum A_i - \epsilon^2 \sum B_i x_i\}/\epsilon^2 \tag{10}$$
$$= n - 2.$$

Substituting this mean value for the value of χ^2 on the left-hand side of Eq. 9, one obtains Eq. 3 of the text.

This result means that the number of *degrees of freedom* of our problem of linear regression of the y_i on the x_i is $n - 2$, that is, it is less by 2, than the number of observations. This can easily be understood: The linear regression is geometrically represented by a straight line drawn, as well as possible, through the n points, y_i, x_i, in the $y - x$ plane. Two of these points may be arbitrarily selected to determine an approximate straight line. This leaves the remaining $n - 2$ points to fit the straight line. If n were equal to 2, the straight line would be completely determined. Its degree of indeterminacy is therefore measured by the excess of n over 2.

In the same way, only with some more extensive computation, it can be shown that:

$$\overline{\chi^4} = n^2 - 2n;$$

also that

$$\text{var } (\chi^2) = \overline{\chi^4} - (\overline{\chi^2})^2 = 2(n - 2),$$

which is as should be. In the derivation of these formulae, we have used the fact that the mean values of the products of the y of the third order all vanish, whereas the mean values of the following products of the fourth order do not vanish:

$$\overline{y_i^4} = 3\epsilon^4, \quad \overline{y_i^2 y_k^2} = \epsilon^4 \qquad i, k = 1, 2, \cdots n \ (i \neq k).$$

All other products of the fourth order vanish.

It is easy to generalize the above derivation. If, for instance, the observations, y, have different weights, we need merely replace the unweighted sums by weighted sums, for example, $\sum x_i$ by $\sum w_i x_i$, etc. If, moreover, the independent variables, x, are also subject to errors, one uses the method of the Lagrange multipliers, as explained in Deming (4), pp. 49–58. However, one should, even in such a case, express the y by a regression on the x and not reverse the correlation by making y the independent variable and express x by a regression on y. This has been done by some authors and has even been practiced in the design of the commercial log-probability charts which present the percentages as the abscissae, and the class end-points, x, as the ordinates. This is not justified and is tantamount to confounding cause and effect. The choice of the independent variable is made not for practical reasons such as subjection to errors, but as a consequence of a physical theory, for example, the law of growth (1).

If the regression is not linear in the parameters, a, b, one must use successive approximations, as explained in the text. Starting with a first approximation, indicated by single primes, one has the equations:

$$y_i = f_i' + \alpha' \frac{\partial f_i'}{\partial a'} + \beta' \frac{\partial f_i'}{\partial b'} + \delta_i''$$

from which are derived the Normal equations:

$$\alpha' \sum w_i \frac{\partial f_i'}{\partial a'} \frac{\partial f_i'}{\partial a'} + \beta' \sum w_i \frac{\partial f_i'}{\partial a'} \frac{\partial f_i'}{\partial b'} = \sum w_i \delta_i' \frac{\partial f_i'}{\partial a'}$$

$$\alpha' \sum w_i \frac{\partial f_i'}{\partial b'} \frac{\partial f_i'}{\partial a'} + \beta' \sum w_i \frac{\partial f_i'}{\partial b'} \frac{\partial f_i'}{\partial b'} = \sum w_i \delta_i' \frac{\partial f_i'}{\partial b'}. \qquad (8a)$$

They replace the above equations, (8). Using the following abbreviations,

$$A_i' = \sqrt{w_i} \frac{\partial f_i'}{\partial a'}, \qquad B_i' = \sqrt{w_i} \frac{\partial f_i'}{\partial b'}, \qquad E_i' = \sqrt{w_i} \delta_i',$$

one obtains:

$$\chi''^2 = \sum w_i \delta''^2 / \epsilon^2 = [E'E'] - \alpha'[E'A'] - \beta'[E'B']\}/\epsilon^2, \qquad (9a)$$

which replaces Eq. 9. Here, the Gaussian symbols, $[EE] = \sum E_i E_i$, etc., have been used. We shall use Eq. 9a, with $n - 2$ as the value for χ''^2, as explained in the text, Eq. 3.

Equation 9a can be used for a short computation of the final Chi-Square without the direct calculation of the residuals, δ''. There is, however, an important exception, if the weights depend on the parameters, a, b, and, therefore, are different from one approximation to the other. The second approximation is then conducted with the weights, w', derived from the first approximation. In the final test, however, the

weights, w'', must be used, which correspond to the final values. Such a case will occur in our applications, since the Pearson weights, Eq. 7, depend upon the parameters. For this reason it is likewise impossible to use Eq. 9*a* as a "final control" of the calculations, as suggested by Whittaker and Robinson (3), p. 234, by comparing the value of the Chi-Square derived from (9*a*) with that obtained by direct computation of the residuals, δ''.

Note that we use the standard errors for the parameters which are the square roots of the variances and, as is well known, are not linked with any probability. Therefore, they do not provide *confidence limits* in the sense used in statistics. If the distribution of the parameters were known to be Normal, there would, of course, be a probability of 68 per cent that the fluctuations of the parameters keep within the limits of their standard errors. But, in case of a frequency distribution, the Normal distribution does not even hold rigorously for the individual observations and their errors, let alone for the errors of the parameters derived from the former. However, if approximate confidence limits are desired, linked with a fixed probability of 68 per cent, it is safe to say that they should be somewhat larger than our limits of the standard errors of the parameters, depending, of course, on the numbers, N and n, involved in their estimation.

Note also that throughout this paper we use as limits of the error band for observations and parameters a width of $\pm$ *one* standard error and not *two* or *three* standard errors as is sometimes done. The use of $\pm$ one standard error is necessitated by the definition of the sum of the squares of the residuals when weights are used and by the corresponding definition of the Chi-Square. However, as a criterion that two values are both within the limits of the error band, we use, as already stated, a distance of *two* standard errors. This is no contradiction to the above statement: As a matter of fact, both values will lie within the width of $\pm$ one standard error if their distance does not exceed two standard errors.

In case of a frequency distribution, the method of least squares must be replaced by the method of Chi-Square Minimum. However, the foregoing discussion of this Appendix still applies provided the number of degrees of freedom is taken as $n - 3$ instead of $n - 2$. (Appendix III.)

APPENDIX III

The Multinomial Distribution and the Chi-Square Distribution

We shall assume that the computed frequencies and their errors obey a multinomial law. This is a generalization of James Bernoulli's Binomial Law, which was mentioned earlier (1).

Accordingly, the probability that class-frequencies, $f_1, f_2, \cdots f_n$ be

observed is given by

$$\frac{N!}{f_1!\,f_2!\cdots f_n!}\left(\frac{f_1'}{N}\right)^{f_1}\left(\frac{f_2'}{N}\right)^{f_2}\cdots\left(\frac{f_n'}{N}\right)^{f_n},$$

where, of course, $\sum f = \sum f' = N$. This equation presumes nothing else than the frequency definition of probability: If f_1' is the computed expected frequency for Class 1, then the probability of the occurrence of one grain among N grains which belongs to Class 1 is given by f_1'/N The combined probability of the occurrence of f_1 grains in Class 1, of f_2 grains in Class 2, etc., and of f_n grains in Class n is then given by the above expression, where the multinomial coefficient [9] takes care of the many possibilities of placing the N grains into the n classes: for example, placing the first grain into Class 1 and the second into Class 2 or placing the first grain into Class 2 and the second into Class 1, etc., etc.

Similarly to the procedure discussed earlier (1), we may replace the multinomial coefficient by the following Stirling approximation, *provided none of the observed class-frequencies is small compared with N:*

$$(1/\sqrt{2\pi})^{n-1}\sqrt{\frac{N}{f_1 f_2\cdots f_n}}\,\frac{N^N}{f_1'^{f_1}f_2'^{f_2}\cdots f_n'^{f_n}}.$$

Next, we introduce the errors,

$$\delta_1 = f_1 - f_1',\ \delta_2 = f_2 - f_2',\ \cdots\ \delta_n = f_n - f_n'.$$

Finally, we substitute for the observed class-frequencies, f, from these equations and develop the resulting expression logarithmically into a Taylor series. We obtain:

$$\left(\frac{f_1' + \delta_1}{f_1'}\right)^{f_1'+\delta_1}\left(\frac{f_2' + \delta_2}{f_2'}\right)^{f_2'+\delta_2}\cdots\left(\frac{f_n' + \delta_n}{f_n'}\right)^{f_n'+\delta_n}$$
$$= \exp\{-1/2(\delta_1^2/f_1' + \delta_2^2/f_2' + \cdots + \delta_n^2/f_n') + \cdots\},$$

that is, the probability of the simultaneous occurrence of errors, δ_1, δ_2, $\cdots\ \delta_n$, is approximately expressed by the product of n Normal frequency functions. The respective means of these functions are zero and their respective standard deviations are:

$$\sigma_1 = \sqrt{f_1'},\ \sigma_2 = \sqrt{f_2'},\ \cdots,\ \sigma_n = \sqrt{f_n'}. \qquad (7)$$

This holds, however, only if we can terminate the logarithmic Taylor series with the terms of the second order. Therefore, it is necessary that, *in addition to the observed class-frequencies, none of the computed class-frequencies be small.* At the same time, the differences, $\delta = f - f'$, between both frequencies must be small compared with them. (Consequently, δ should be small compared with f'; it need, however, not be small compared with $\sigma = \sqrt{f'}$.)

[9] That is, $\frac{N!}{f_1!f_2!\cdots f_n!}$.

Under these two conditions, we may replace the multinomial distribution by a multivariate Normal distribution of the errors in a frequency distribution. However, this Normal distribution is different from that assumed by the principle of the least squares: The errors are no longer independent of one another but bound by Eq. 5:

$$\sum \delta = 0.$$

Collecting our results, we find, for the joint probability that class-frequencies, $f_1, f_2, \cdots f_n$, be observed corresponding to the computed class-frequencies, $f_1', f_2', \cdots f_n'$, the expression:

$$(1/\sqrt{2\pi})^{n-1} \sqrt{\frac{N}{f_1 f_2 \cdots f_n}} \exp(-\chi^2/2).$$

Here,

$$\chi^2 = \sum \delta^2/\sigma^2 \tag{6}$$

and

$$\sigma = \sqrt{f'}. \tag{7}$$

The frequencies are bound by the equation,

$$\sum \delta = \sum (f - f') = 0, \tag{5}$$

and must be large, the observed as well the computed ones.

A new all-purpose exposure suit developed by the Air Materiel Command's Aero Medical Laboratory will keep a ditched airman afloat in water or protect him from exposure on land. The basic suit, fashioned from rip-stop nylon double coated with water impervious chloroprene weighs only 7½ lb., is equipped with valves to permit plug-in for G and altitude suits. A sponge rubber closing around the neck, water sealed zipper front closing, and neoprene rubber straps at wrists effectively seal the suit against water entering these openings. Production models of the suit will be dyed high-visibility yellow.

Color problems in camouflage are being studied by the Engineer Research and Development Laboratories at Fort Belvoir, where spectral reflectance and wave length of colors are found to be as important as the color itself. Projects involving surface appearance include camouflage coloration of combat uniforms and appearance characteristics of camouflage nets and garnishing materials. Plastics are being investigated as a replacement for burlap, with embossing, flocking and creping being investigated as means of eliminating surface gloss.

Chart for Microchemical Analysis.—Publication by the Geological Survey of a reference chart designed to facilitate systematic microchemical analysis was announced recently by Secretary of the Interior Oscar L. Chapman.

Developed by Geologist Fred M. Chace for ready reference in the laboratory, the chart may be pasted on stiff cardboard and hung near the microscope, making it unnecessary to refer constantly to a cumbersome text book during analytical work. In general, the chart follows the procedures described by M. N. Short in Geological Survey Bulletin 914, 1940, reprinted in July 1948, with slight additions and corrections.

Whenever possible opaque minerals should be identified by the usual methods of determinative mineralogy; that is by noting color, hardness, specific gravity, cleavage, crystal system and other physical and chemical properties. In polished sections under the microscope, hardness, polarization, and etch tests are usually sufficient to identify the mineral. If in doubt, microchemical tests for one or more elements or more elements or specific tests for particular minerals can be applied.

If none of these methods gives satisfactory results, then the systematic procedure of qualitative microchemical analysis can be followed, using the new chart. First test consists of scratching the unknown with a Sharp's No. 6 needle to determine whether the mineral is soft or hard. Then the more complicated tests should follow as outlined. Though the chart is a quick guide it is not intended to replace Short's bulletin for details on techniques and procedures.

This chart is available as a sale publication at 15 cents a copy through the Chief of Distribution, Geological Survey, Washington 25, D. C.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

GLASS VALVES FOR VACUUM CONTROL

One of the problems encountered in vacuum practice is the ready and accurate control of gas flow into or through the system. It is important that the device used for such control should not be a source of contamination and should not liberate unwanted gases. The standard stop-cock normally used for this purpose has been found unsatisfactory because its grease seal does not permit application of the high heats necessary for degassing. This results in undesirable contamination while the grease itself is a source of trouble.

To overcome these and other difficulties, as well as to provide an improved seal, R. Forman of the National Bureau of Standards electron tube laboratory has developed a glass-seal type of valve which operates at pressures of the order of 10^{-6} mm. of mercury. The new valve depends on the expansion of glass or metal when heated. This elongation characteristic is utilized to break a glass seal which separates the vacuum system from another of higher pressure. When the source of heat is removed, contraction of the glass reseals the system.

The valve has the appearance of a conventional water condenser with but one side-arm. It is made of 7052 hard glass, chosen because its coefficient of expansion matches that of a Kovar sleeve inserted into the outer jacket. The center tube is sealed at one end and terminates at the other end in a graded seal, permitting the incorporation of the unit into any pyrex system requiring this type of control.

One of the properties of two optically polished surfaces is the enhancement of the intermolecular force which acts to hold the two surfaces in contact. Before incorporation into the system, the concentric tubes of the glass valve are carefully cut with a glass saw, and the surfaces are optically polished to a high degree of flatness. The ends are then placed in contact once again and are held in this position by the molecular force. A forepump is connected to the inner tube, and the unit is pumped to some convenient value of vacuum. With the external pressure and the intermolecular forces holding the cut sections together, the outer jacket is glass-welded with a sharp flame. At this stage, there are two forces on the inner tube: the molecular adhesion and the external air pressure. During the cooling process, contraction adds another force. The total effect of these forces makes the seal vacuum-tight.

* Communicated by the Director.

Under standard operating conditions, the inner tube is at the desired vacuum, and the space between the inner and outer tubes is at approximately the same pressure as the gas to be admitted to the system. The Kovar sleeve is heated to a temperature of about 200° C. by an induction heater, and heat flows from the Kovar into adjacent parts of the outer glass jacket. The over-all expansion of the outer tube exerts a force which opens the cut in the inner tube. When the desired quantity of gas has entered the system, the induction heater is turned off, and the resulting contraction of the outer jacket once again seals the system.

In operations requiring the admission of special gases to the system, a gas bottle is sealed to the sidearm. Several variations of this particular design and use of the glass valve are possible. Instead of the Kovar sleeve inserted into the outer jacket, the unit can be made entirely of pyrex, and the heat necessary to cause an expansion supplied by an external heater.

Tests have shown that this valve makes possible a continuously variable control of the rate at which gas can be admitted to an evacuated system. Complete data are not yet available for establishing a lower limit for use as an adjustable leak. However, initial measurements indicate that leakage through the valve into a one liter volume will raise the gas pressure by as little as several tenths of a micron in one hour. On the basis of this information, another application for this device would be as a calibrated leak for a mass spectrometer leak detector.

ELECTRONIC COUNTING OF PAPER MONEY

A fully automatic electronic machine for counting worn-out paper money has been designed and developed by H. M. Joseph and Carroll Stansbury of the National Bureau of Standards for the Department of the Treasury. The NBS Electronic Currency Counter counts 30,000 notes per hour. Twenty-five of these machines will be placed in service soon by the Treasury, replacing the present hand-count system and saving about $250,000 annually.

Although new paper money has been machine-counted for many years, mechanical handling of worn-out notes has until now been a perplexing problem. Money returned to the Treasury is in the form of packets of 100 notes cut longitudinally into half-notes. About 8 tons of currency are turned in daily for redemption. The bulk of the returned money (about 5 million dollars' worth) consists of one dollar bills, constituting about 80 per cent of all the pieces of paper currency received for redemption. The notes are limp, wrinkled, and difficult to handle; occasionally single torn notes are taped together. The variable condition of returned money has required tedious counting by hand. The need for an effective mechanical means that would

combine both rapid counting and accuracy prompted the Treasury Department to initiate a developmental program at the Bureau directed toward machine counting.

The NBS Electronic Currency Counter counts the packets of notes and automatically rejects those with more or less than 100. Packets of stapled half-notes are placed on an inclined trough. A metal finger pushes the bottom packet into the machine. The stapled end is grasped by metal jaws and the packet is wrapped tightly around a spindle, spreading the outer edges of the notes against a friction band. As the spindle rotates, the notes clear the friction band and are separated by a jet of air.

The counting is achieved by using an electronic sensing device consisting of a beam of light and a phototube system. As the notes are unfurled, interrupting the beam of light, the phototube senses the interruptions of the beam. Impulses from the phototube are fed to a binary counter which tallies the individual impulses. The sum of these impulses is used to actuate a sorting vane: if the count is 100, the sorting mechanism automatically sends the packet into an acceptance bin; if the count is more or less than 100, the packet is automatically ejected into a reject bin. Except for filling the trough with packets to be counted and removing those already counted, the machine operates unattended. In case of mechanical or power failures, it will shut itself off.

Description of Machine

For flexibility, each machine is a separate, completely self-contained unit, mounted on casters. For ease of maintenance, the counter is composed of replaceable, completely interchangeable subassemblies.

At the beginning of each counting cycle, a feeder mechanism pushes each packet endwise from the inclined trough into the opening between the spindle jaws. When a packet reaches this position, an electrical limit switch is actuated, which causes the spindle jaws to clamp and rotate, rolling the packet around the spindle for counting. The jaws then release and the packet is cleared from the counting head by an additional revolution of the spindle. The released packet falls on a sheet-metal sorter vane which has been tilted to the correct or the reject position by an electromagnet responding to the count. The spindle jaws, together with an electromagnet which operates them, are carried in a rotating element of a turntable assembly mounted on top of the unit.

A high degree of reliability is attained in the NBS Electronic Currency Counter through the use of the light beam counting technique. An optical assembly including a lamp, condensing lens, and two small adjustable mirrors, deflects a light beam across the path of the sheet ends as they are blown by the air jet.

When the technique is applied to the worn half-notes, certain critical design requirements must be observed. The light beam must have an appreciable cross-section (approximately 1/8 by 1/4 in.) to avoid false operation by stray bits of paper and must be placed at such an angle as to be reliably interrupted only by single sheets.

Mechanical details of the pusher which holds the end of the friction band against the contour of the rolled-up sheets are rather critical. It was found that a knife-edged bearing offered the best solution to the problem of releasing only individual sheets. Another critical design feature was the position of the air-jet nozzle relative to the end of the friction band. Its opening was placed as close as possible to the contour of the surface and set to produce an airstream tangential in direction. This proved to be the only arrangement which would give the proper swinging motion to the individual ends of the half-notes as they passed the light beam.

The electronic counter is relatively conventional. A cathode follower applies the phototube impulses to a trigger circuit for conversion to sharp, uniform-amplitude pulses. These pulses are, in turn, applied to a binary electronic counter. The 8-stage counter is set to indicate the desired count for a correct packet of 102 (100 half-notes, plus the wrapper sheet on each side). In ordinary production use by the Treasury, the actual numerical count for each packet is not of interest, but for maintenance purposes the total is indicated by a series of neon lights located just below the output chutes.

In the event a packet of half-notes tangles around the spindle, a limit switch stops the automatic feed until the machine is cleared by hand. Other limit switches stop the machine after it runs out of packets and interlock the motion of the turntable with that of the feeder mechanism.

The device is "fail-safe" in the sense that a very large percentage of possible failures in the equipment are such as to cause all packets to be rejected. This would be quickly noticed by attendants and result in the particular machine being taken out of service. The less likely failure, in which all packets are accepted, would have the same result.

Machine Performance

The most important factor in the counting effectiveness of the machine is the condition of the money to be counted. Thus, causes of erroneous low count include extensibility of half-notes due to wrinkles, folded corners which interfere with individual release of the sheets, folded or short sheets, and adherence of adjacent sheets due to taping of half-notes with gummy tape, cancellation punching with dull dies, or other causes.

A study of optimum spindle speeds was made to determine the effect of varying speeds on accuracy of count. The advantage of increased accuracy at low operating speeds was compared with the cost of a greater number of machines. As a result of the study, the spindle speed adopted is 15 rpm., but this is subject to revision upward on the basis of anticipated large-scale tests which will accompany early production operation. Since the spindles are belt-driven, revision of speed is a simple matter.

Basic Counting Methods

The Bureau's first attack on the problem of counting money employed a transverse scanning method—similar to television scanning—across the clean-cut edges of piled and cut half-notes. It was immediately recognized, however, that the condition of the sheets resulted in irregularities and gaps between them, which made a single scan not sufficiently reliable. Subsequent studies of repeated scanning showed that the ratio of the width of the thinnest gap (corresponding to one note) to the total thickness of the pile of sheets was of such magnitude as to require resolution beyond that available with the best optical system.

These difficulties led to attempts to find ways of physically separating the sheets sufficiently to permit reliable actuation of a sensing apparatus. This was finally accomplished by clamping one end of a packet of half-notes between the opposing faces of a spindle structure and wrapping the free ends around the spindle in a roughly circular contour. This scheme separated the ends of successive sheets by a sufficient interval.

The first sensing apparatus tested was a contact stylus of a vibration pick-up. As each note passed under the stylus, an electrical impulse was generated and the count noted. This system would have been useful for counting money in good condition but there was some doubt as to its effectiveness with old and wilted half-notes. Test operations, involving the counting of several thousand packets of half-notes, each containing 100 notes, resulted in a high count on about 5 per cent of the packets (a positive error of about 5 parts per 10,000). These results led to the abandonment of the vibration pickup method in favor of the light-beam technique.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, MAY 16, 1951

The May stated meeting of The Franklin Institute was held in the Lecture Hall on Wednesday evening, May 16, with approximately 120 persons attending. The President called the meeting to order at 8:15 P.M., and stated that the minutes of the April meeting were printed in full in the May issue of the JOURNAL. There being no corrections or additions, the minutes were approved as printed.

The President summarized the Annual Report of the Board of Managers for 1950 and announced that the complete Report would appear in the July JOURNAL.

In his report, the Secretary stated that Sir James Chadwick, Franklin Medallist for 1951, had been nominated by the Board of Managers for Honorary Membership in The Franklin Institute. In accordance with the provisions of the By-Laws, a vote was called for, and all votes heard having been in the affirmative, Sir James Chadwick was elected to Honorary Membership.

It was reported by the Secretary that the following elections to membership were recorded for the month of April:

Active	40
Associate	31
Student	8

The total Institute membership as of 30 April was 6,029.

The Secretary presented the following announcements concerning our Museum:

1. Projects of the Eastern regional winners of the Ford Motor Company's 1951 Industrial Arts Awards which are selected on June 30 will be on public display at the Institute daily except Monday from July 3 to 15.

2. The National Junior Chess Tournament will be held here from July 23 to July 28.

3. From August 4 until September 9, the Institute will feature the Baltimore & Ohio Model Railroad Exhibit.

4. Now in its second month, the current planetarium show, "How Will the World End?," can be seen daily except Mondays through the end of May. The June show entitled "Stars in Your Eyes" is the story of the constellations.

A few words were said by the Secretary about the recent organization of the National Asphalt Research Center by The Franklin Institute Laboratories for Research and Development.

The President told of the annual Charles Day Lecture, and introduced Mr. Henry B. Bryans who gave the Lecture for 1951. His absorbing talk, entitled "Dedication to Public Service," was received with approbation by the audience, and Mr. Bryans was tendered a rising vote of thanks as the meeting adjourned.

HENRY B. ALLEN
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, May 9, 1951.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MAY 9, 1951.

DR. RUPEN EKSERGIAN in the Chair.

The following report was presented for final action:

No. 3235: Levy Medal.

This report recommended the award of the Louis Edward Levy Medal to Albert Charles Walker, of Short Hills, New Jersey, "In recognition of his outstanding paper, 'Growing Piezoelectric Crystals,' appearing in the December, 1950, issue of the Journal of The Franklin Institute."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS May 16, 1951

ACTIVE FAMILY

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Gustav T. Reich
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Franklin H. Wells

NECROLOGY

William H. Ballman '43
Colonel W. P. Barba '44
R. W. Eichenberger, Jr. '48
J. Logan Fitts '12
Mrs. Isaac Gerstley '36
Robert G. Goodyear '49
Mrs. George R. Henderson '38
Charles W. Rivise '30

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

MOON, PARRY AND DOMINA EBERLE SPENCER: Interflectance Calculations for Various Luminaires.

SILBERSTEIN, LUDWIK: The Effect of Gradual Light Absorption in a Film of Non-Negligible Thickness.

WEINBERG, LOUIS: Solutions of Some Partial Differential Equations (with Tables).

HIGGINS, THOMAS JAMES: Solution of Boundary Value Problems by Multiple Laplace Transformations.

HUNICKE, A. BYRON: The Valved-Tremie Applied to Subaqueous Concrete Structures.

RAMSEY, W. E.: The Slow Discharge in a Non-Self-Quenching Geiger-Mueller Counter.

ZETROUER, W. F. AND WILLIAM J. KESSLER: A Note on the Similarity of Certain Atmospheric Waveforms.

PATAI, IMRE AND MARTIN A. POMERANTZ: Contact Potential Differences.

KRZYWOBLOCKI, M. Z.: On the Foundations of Certain Theories of Turbulence.

SPENCER, DOMINA EBERLE AND VINCENT J. STAKUTIS: The Integral-Equation Solution of the Daylighting Problem.

SYMONDS, P. S.: Recent Progress in the Plastic Methods of Structural Analysis.

KNOWLES, T. AND A. H. TAYLOR: Spectral Radiation Involved in Photoreactivation of Ultraviolet-irradiated Cultures of Micro-organisms.

MUSEUM

During its more than one hundred and twenty five years of operation, it is understandable that the methods adopted by the Institute for attaining its objectives should have been subjected to no small amount of amendment. As soon as one activity fostered by the Institute could be transferred to another organization willing and able to support it to better advantage it was relinquished and a fresh field in which the industrial community could be served was sought. Vocational schools and exhibitions were among the Institute's most carefully nourished pioneer activities. The former passed under the control of the city authorities when technical education was admitted to the curriculum of the public schools, and American industries grew up to be robust enough to support their own exhibitions. The removal of these two activities left a vacuum in the Institute's functions which could not be readily filled. However, the passage of time has brought about the opportunity to make a happy combination of these two old activities in a way which fits into the Institute's program.

The third Annual Science Fair has just been held, in which the students of local schools displayed an impressive demonstration of their interest in scientific subjects. It is quite apparent that these fairs, which are co-sponsored by the *Philadelphia Inquirer*, have become established events among the young people in this area. But they do not constitute the only events of the kind which appeal to technically minded young people as opportunity to display their talents. From July 3rd to the 15th we are to have the opportunity to see another of these exhibits in a display of the entries for the Industrial Arts Awards, sponsored by the Ford Motor Company.

In the annual Science Fairs the ground covered has been comprehensive, embracing any scientific study. Industrial Arts Awards are, as the title implies, somewhat more restricted but, none the less, are broad enough to have wide appeal. The general purpose is expressed by Mr. Henry Ford II in these words: "Despite the high degree of specialization that characterizes modern mass production methods, there remains a great need for fine craftsmen throughout American industry. Just as important is the need for pride of performance on the part of each individual regardless of his task. Industrial arts and trade and industrial courses in our junior and senior high schools are doing much among our young people to kindle a greater appreciation of hand skills and a spirit of pride of performance. We believe that these activities are extremely worthwhile. We hope that through the Industrial Arts Awards we can provide encouragement and recognition for students and for teachers who are doing outstanding work in this field."

Entries from all over the countries for these awards are being assembled at three regional headquarters of which The Franklin Institute is one. The entries from the eastern States will be shown in Franklin Hall during the period mentioned earlier.

The nature of the entries to be expected may be gathered from the classification of subjects, which includes wrought metal, pattern making and moulding, machine shop, wood, plastics, electrical, mechanical drawing, architectural drawing, and printing. Prize winning entries at the regional exhibitions will be assembled for final judging to select national prize winners. Visitors to the display in Franklin Hall, therefore, will have the opportunity to see the best craftsmanship our young people are producing and, from such samples as we have seen at the Science Fairs, the standard is sufficiently high to encourage high hopes that skill in the industrial arts is not likely to degenerate in this country.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledge and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open 9 A.M. until 5 P.M. on Mondays, Tuesdays, Wednesdays and Fridays; 2 P.M. until 10 P.M. on Thursdays; and 9 A.M. until 12 noon on Saturdays.

RECENT ADDITIONS

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SMART, WILLIAM MARSHALL. Some Famous Stars. 1950.

AERODYNAMICS

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MAGNUS, RUDOLF. Goethe as a Scientist. 1949.

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MACKENZIE, F. W., Ed. Screen Process Printing. 1951.

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BOOK REVIEWS

CLASSICAL MECHANICS, by H. C. Corben and Philip Stehle. 388 pages, illustrations, 14 × 22 cm. New York, John Wiley & Son, Inc., 1950. Price, $6.50.

The presentation of classical mechanics can have a two-fold approach, as in this book: first, a development of over-all principles from a macroscopic viewpoint which are particularly valuable in the many problems in applied physics and engineering; and second, emphasis on those points that can be effectively carried over to microscopic phenonema which can only be effected from the quantum aspect. Along this carry-over are certain mathematical techniques that can be approached in a more realistic way through classical mechanics serving as a scaffold in later quantum analysis. Thus the extension of generalized coordinates to normal modes of vibration along with the matrix and tensor notation, is particularly useful for the conception of eigenfunctions and the orthogonal properties of these functions, though their interpretation to quantum as probability-wave functions governed by quantum energy levels, is quite different from that of the classical resonant frequency spectrum.

A special feature of this book is the introduction at the offset of the Riemannian geometry and transforms. The book opens with a review of some orthodox problems in analytical terminology. It then proceeds with a discussion of the kinematics of dynamical systems in terms of Riemannian geometry and tensor notation showing the relation of coordinates in the transform to different coordinate systems and the distinction between contravariant and covariant vector components, along with the significance and notation of the metric tensor. Finally the Christoffel notation for the contravariant components for acceleration are developed. This is followed with a brief outline of vector algebra.

The development of the Lagrangian equations in terms of generalized coordinates in a Riemannian space permits the direct extension of the generalized equations of motion in terms of the Christoffol notation and metric tensor.

In connection with the association of Lamellar fields with the Lagrangian equations of motion, expressed in terms of the Lagrangian function and equated to zero, the authors point out that a generalized force may still be added, provided the energy function for the generalized force depends only on the coordinates and velocity components. This relates the electromagnetic field forces in a dynamical system in terms of the vector potential directly developed from the Lagrangian function.

From the algebra of linear vector space and matrices, the authors develop the necessary background for vibration systems, where normal modes of vibration are in turn introduced for systems of several degrees of freedom. The authors coordinate the eigenvalue problem and eigenvectors in this discussion. A fairly comprehensive discussion is also given to molecular space lattice vibration systems and couplings between the spin motion.

The remaining portion of the book covers a good presentation of the Hamilton-Jacobi Equation, Poisson Brackets and Contact Transformation theory. Particular stress is given to the Hamiltonian function. The final chapter is devoted to the Special Theory of Relativity and the Lorentz transformation, including the electrodynamics of the motion.

From a review of the contents, this book appears of particular value for the transition to atomic physics and quantum mechanics. It is questionable, however, if the work can be considered as adequate in its general treatment of dynamics. Thus non-holononic systems are disregarded, variation methods are limited while the interesting physical relations in gyroscopic phenonema could be covered by suitable problems. Further, a direct development of the equations of motion in terms of the Christoffel notation, using only the principle of virtual work, would emphasize an explicit form independent from the derivation through the Lagrangian functions. It would appear desirable to have maintained a more simple and parallel development, for the understanding of dynamical principles, using simple analytical expressions in terms of generalized coordinates rather than the more confusing derivation in

terms of covariant and contravariant components of vectors and tensors as in the derivation of the Lagrangian equations.

The book, however, gives a new approach through the classical phase of mechanics, by introducing new mathematical techniques, materially aiding in the transition to quantum mechanics. As such, it is a valuable contribution.

RUPEN EKSERGIAN

THE PROPERTIES OF ASPHALTIC BITUMEN, edited by J. Ph. Pfeiffer. 285 pages, tables and illustrations, 17 × 26 cm. New York, Elsevier Publishing Co., Inc., 1950. Price, $6.00.

This book presents the findings and conceptions of several asphalt pioneers associated with the Royal Dutch Shell Laboratories in Amsterdam. The collaborators include, besides Dr. Pfeiffer, H. Eilers, J. W. A. Labout, R. N. J. Saal, M. C. Siegmann and H. W. Slotboom, men who have contributed substantially to our knowledge of asphalt. While the book briefly describes processing methods and properties of products containing asphalt, its emphasis is on the nature and fundamental properties of this extremely complex material. Discussions of chemical composition, colloidal aspects, thermal and electrical properties, solvency, reactivity and adhesion are particularly valuable to the American Research Worker because they present much information not hitherto available in this country. The section on rheological properties, while interesting in that it presents the conceptions of some of the foremost scientists in the field, contains less that is novel to American workers.

The explanations in the book are quite lucid since, as might be expected from the fact that its authors are research men, they are quite free of dogmatism. The authors do not hesitate to point out where there is a lack of knowledge of the fundamental nature of asphalts. This book is highly recommended to research men but is in no sense a compendium of facts, which, like Abraham's *Asphalt and Allied Substances*, would appeal to less specialized practitioners. In the work of Royal Dutch Shell Laboratories it has ever been necessary to deal with both the theoretical and practical aspects of asphalt technology, and the relations between them. The value of the book is enhanced by the fact that the two aspects are well integrated.

EDMUND THELEN

WATER TREATMENT FOR INDUSTRIAL AND OTHER USES, by Eskel Nordell. 526 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Co., 1951. Price, $10.00.

The treatment of water is a rather involved and complicated process, but the author has drawn together in a complete and well organized volume most of the information available on this subject. This book is much more complete than many of the works which have appeared on this subject.

Mr. Nordell writes very effectively and deals with water sources, softening, coagulation and sedimentation, boiler feed waters, cooling waters and other aspects of industrial water conditioning. A large number of notes placed throughout the book are frequently from the author's personal experience in the water treatment field. They are interesting and often entertaining.

While the book is well illustrated, the equipment shown is exclusively that manufactured by one company. A wider coverage of the equipment manufacturers in this field might have been desirable. Several statements in regard to the use of sodium silicate for corrosion inhibition are not in accordance with current practice and one would question the point made in regard to the effect of silica in causing "caustic embrittlement."

The bibliographies at the end of the chapters are adequate and the comprehensive tables in the appendix are very useful to one actively engaged in this field. The book should be a valuable addition to the library of those interested in water treatment.

A. B. MIDDLETON

THE ORIGIN OF THE EARTH, by W. M. Smart. 239 pages, illustrations, 14 × 20 cm. Cambridge, University Press, 1951. Price, $2.75.

Two topics which have been the subject of philosophical speculations among scientists everywhere deal with the origin and end of the earth. In the case of the latter we have no

means, as yet, of extrapolating so far in the future and so this must remain a mystery. As for the origin of the earth, the beginning of civilization marks the beginning of man's efforts to solve the riddle of the world's beginning.

In a superbly written book, a keen and prolific writer in astronomy, Dr. W. M. Smart, Regius Professor of Astronomy in the University of Glasgow, has covered the thoughts of mankind on this subject.

The book is logically divided into three sections. The first deals with the parentage of the earth and the solar system. He shows there is an organic unity in the solar system and it is inconceivable that its members did not have a common ancestor. In this section is found an excellent description of the solar system.

Part 2 deals with the time scale of the event. Here the methods of modern science used to determine the age of the earth are disclosed. This is a good, factual history of the subject. In this section are found the details of the various nuclear cycles in a description of which the author indulges in some curious statements. For instance, on page 125 is found "there is also evidence of the existence of a third isotope of (hydrogen) atomic weight 3." He displays extreme caution in some of his statements which are wholly unwarranted. Were it not for the existence of tritium which was first manufactured in September 1948 the assembly of the hydrogen bomb, which we are told is now in progress, would be an impossibility.

The last section deals with the mechanism of the creation of the earth. Here once more is collected a vast amount of historical information showing the evolution of our present day theories.

The *Origin of the Earth* is a highly instructive book in which may be found the answers to many of the provocative questions the reader may ask about the earth's beginning. The answers, to be sure, are speculative and, while there is no guarantee that what has been written today will not be outmoded tomorrow, it still represents the current integrated thoughts of the scientist.

I. M. LEVITT

PETROLEUM GEOLOGY, by Kenneth K. Landes. 660 pages, illustrations and plates, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1951. Price, $10.00.

At a time when interest in oil and oil sources is quite high, such a book as *Petroleum Geology* is particularly opportune. Partly because of a minimum of technical geological terms and partly because of the timely inclusion of maps and geological profiles (to the delight of any one who has had to search for an atlas while reading a book with geographical references but no maps!), the book may be read and easily understood by layman and geologist alike. Writing in a fluent style, Professor K. K. Landes of the University of Michigan has prepared a comprehensive volume on the aspects of exploration, exploitation and the origin and evolution of petroleum.

Introducing the subject with a description of the petroleum geological professions, including a brief history of the petroleum industry, oil finding, and present trends in petroleum geology, Professor Landes launches into the first of the three main divisions of his book. Part I, the techniques of the petroleum geologist, is divided into exploration for petroleum and exploitation after discovery. The emphasis is on the exploration which, fundamentally, concerns the geologist more than the exploitation. All the known methods for locating oil are quite thoroughly delineated.

Part II, the geologic occurrence of petroleum, is divided into the physical and chemical properties of oil and gas; the origin, evolution and migration of oil and gas; and the accumulation of oil in reservoir rocks and traps. In Part III, the present and future oil supplies, the distribution of oil throughout the world are discussed. Since it occupies about half the book's 660 pages, an appreciable amount of information is contained in this section. An appendix of three pages contains a list of recent publications on the literature of petroleum geology.

E. W. HAMMER, JR.

GEOGRAPHY IN THE TWENTIETH CENTURY, edited by Griffith Taylor. 630 pages, illustrations, maps, 14 × 22 cm. New York, Philosophical Library, 1951. Price, $8.75.

The question as to how far the physical features of the earth affect man is an old one. It is also one to which no final answer has so far been given. What is geography? The modern geographer's conception of his field is very far removed from the restricted view of physical features. One of the definitions offered in this book declares it to be anything the geographer does. That is a frame of reference which permits a generous latitude of thought, and the contributors to this volume have taken advantage of it. The expansion of the study of geography during the past fifty years has been so significant that the time had come to make an adequate survey of all that had been accomplished. This volume was greatly needed, and it succeeds admirably in presenting a comprehensive picture of the subject.

The book is divided into three parts, of which the first deals with the evolution of geography and its philosophical basis. The second part treats with the factors of environment—topography, climate, soil, etc. The final part is concerned with field work, sociological aspects, and geopolitics. The twenty-six chapters are the work of twenty different contributors, each of whom is a specialist in his field. The result is a handsome volume with a vast amount of information on the growth, techniques, and aims of geography.

Treatment by specialists is undoubtedly the best means to approach the subject, for it embraces so many fields that very few geographers are qualified to make the wide excursions necessary. But while specialists might be trusted to give a competent handling to their subjects, those chosen by Dr. Taylor have contrived to give their special fields a touch of distinction which makes this book a work of outstanding merit.

Where there is so much that is worthy of praise it may seem invidious to select any individual chapter for special mention, but Bowman's "Settlement by the Modern Pioneer," Fawcett's "Geography and Empire," and Taylor's "Geopolitics and Geopacifics" should be read by everyone who is interested in the problems of this troubled world. These chapters might also be recommended to teachers of geography as indication of the means by which many more students could be attracted to the subject.

T. C.

THE PRINCIPLES OF CLOUD-CHAMBER TECHNIQUE, by J. G. Wilson. 131 pages, illustrations, 14 × 22 cm. Cambridge, University Press, 1951. Price, $2.75.

Written primarily for those who use or plan to use the cloud chamber as a research tool, this book is not a guide to the construction of a cloud chamber, although a discussion of constructional materials is included. It is, rather, a description of the mechanics involved in its operation. The production and maintenance of suitable atmospheres are dealt with in chapters on the condensation and growth of drops, ionization in cloud tracks, and the production and maintenance of super saturation. The recording of data is treated in chapters on operation and photography, and counter control. The technique of precision measurement of the recorded data is then discussed at some length and the author concludes with a short discussion of the interpretation of cloud-chamber photographs.

L. SCHIEBER

THE GROWTH OF PHYSICAL SCIENCE, by Sir James Jeans. Second edition, x plus 364 pages, illustrations and diagrams, 13 × 19 cm. Cambridge, at the University Press, 1951. Price, $3.75.

In 1947, shortly after the death of the author, the first edition of this book appeared. The present second edition has been prepared by Mr. P. J. Grant of the Cavendish Laboratory, Cambridge.

A list of books by Sir James, given on the jacket, includes *The Universe Around Us*, *The Mysterious Universe*, the delightful *The Stars in Their Courses* which grew out of a series of talks on the "wireless," *The New Background of Science*, *Through Space and Time*, *Science and Music*, and *Physics and Philosophy*. The earlier *Stars in Their Courses* lists also those masterful works

for which the author is more famous, if not more widely known: These include *The Dynamical Theory of Gases, The Mathematical Theory of Electricity and Magnetism, Astronomy and Cosmogony*, and *Problems of Cosmogony and Stellar Dynamics*. Obviously the author is well qualified to discuss the growth and interrelation of the sciences, especially the physical sciences, from the earliest times to the present.

While the style is simple, some knowledge of algebra, geometry, physics and astronomy on the part of the reader will increase his appreciation of the book, which ties together numerous subjects but cannot explain each one with any thoroughness. Relativity is discussed in eight pages, the quantum theory in ten; the first 288 pages are devoted to the years before 1887. Of eight chapters, only one is devoted to modern physics.

In a book of this sort a choice must be made between completeness and readability and the resulting compromise is arrived at after assumptions have been made concerning the preparation and interests of probable readers. It might appear that in the present book the author leaned over backward with the result that the book may be too crammed with historical detail to interest the lay reader for whom it appears to have been designed. But for the person with some background and training in the physical sciences, and an interest in scientific history and development, the book is excellent.

C. D. CHASE

POSITIVE-DISPLACEMENT PUMPS AND FLUID MOTORS, by Warren E. Wilson. 250 pages, illustrations, 16 × 24 cm. New York, Pitman Publishing Co., 1950. Price, $7.50.

Positive Displacement Pumps and Fluid Motors by W. E. Wilson is one of the first complete books on positive displacement hydraulic machinery. In view of the high degree of development which these components have reached in recent years, it is surprising that there has not been more technical literature published on these devices.

The author does not claim to have written a design handbook nor a complete theoretical treatise on hydraulic machinery. His treatment of the subject is rather elementary and some phases could be amplified and extended to increase the usefulness of the book.

Initial chapters deal with elementary principles of positive-displacement action and the deviations from ideal conditions due to leakage, friction, cavitation, compressibility and other factors. Considerations are also given to various hydraulic pump-motor combinations and some of the more apparent speed-displacement characteristics.

The classification of fluid pumps and motors is covered in some detail. Two broad categories of machines are piston and rotary types. The former includes both axial and radial piston machines while the latter includes various gear and vane types. Pictures of commercially available units illustrate each type.

A section on the application and selection of positive-displacement units is too brief to be of much help in selecting the proper machine for a specific job. For example, no mention is made of the dynamic capabilities of hydraulic motors and means of selecting them in applications where fast response is mandatory.

Performance calculations are illustrated for several types of hydraulic units and test procedures are outlined. A short section on elements of fluid flow is included as background for approximating slip and resisting torque. These lead to expressions for volumetric, torque and over-all efficiency.

A design procedure is outlined which is intended to be a systematic procedure to follow in the design of any positive-displacement machine. Steps are also shown by which the performance of the unit can be estimated. The closing chapters discuss the possibility of employing a positive-displacement hydraulic machine as a clutch and discuss performance characteristics of hydraulic transmissions. A differential transmission which has a distinct efficiency advantage is also discussed. An appendix contains a bibliography of technical literature published on the subject since 1929.

R. H. MARVIN

MATHEMATICAL ENGINEERING ANALYSIS, by Rufus Oldenburger. 426 pages, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1950. Price, $6.00.

Five major topics—Mechanics of Rigid Bodies, Electricity and Magnetism, Heat, Elasticity, and Fluid Mechanics—are discussed in this reference text. Each major topic is further subdivided in order to discuss adequately the fundamental principle prior to applying these principles in illustrative examples. These examples, expressed in the language of engineers, make the book of special interest to the engineering profession. Each major topic is followed by a list of problems pertaining to that topic. An added attraction is the hints and answers given for these problems at the end of the text.

A. D. H.

POWER SYSTEM STABILITY, by Edward Wilson Kimbark. Volume 2, 288 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $8.00.

Practicing engineers in the power field and graduate students studying power system stability will find in Prof. Kimbark's book much new and unusual material which has previously appeared only in the form of technical articles. Circuit breakers and protective relays are basic to power system stability because fast clearing of faults and rapid reclosing of these elements are comparatively inexpensive means of improving the transient stability of power lines, that is, the power which can be transmitted by a system during a fault.

The first of the four chapters in the book discusses the role of power circuit breakers in clearing faults. Detailed descriptions are given of currently available oil and air circuit breakers, followed by sections devoted to: operating mechanisms; arc extinction; ratings and tests; and a sample calculation in which a series of breakers to protect a typical small power system are to be specified.

Protective relays assist in fault clearing because they must select the proper circuits for operation and produce control information rapidly and accurately. As with the previous subject, the author presents principles of constructions, design, and application for a wide variety of different relays, including overcurrent, directional, distance, ground fault, pilot, carrier, apparatus, and bus relays. The basic mathematical theory of the several relay designs is presented by numerous line diagrams.

Not only must relays help clear and reclose faulted circuits but they also must separate synchronous machines which surge violently with respect to one another. A mathematical analysis of surging and out of step operation is presented in the third chapter for a two machine system, considering especially the effects on overcurrent and carrier pilot relays. The illustrative problems worked out in detail help in understanding the principles involved.

The two machine problem is further considered in the fourth chapter which deals with rapid reclosing, a necessary requirement for high transient stability. The equal area criterion is used in the explanation and illustrative problems, which analyze single and three pole switching, the effects of deionization time, and the past performance records of power system stability.

The set of design problems included in each chapter and the rather extensive bibliography provide more than enough material for any intensive graduate course on this subject.

S. CHARP

ADVANCES IN RADIOCHEMISTRY AND IN THE METHODS OF PRODUCING RADIOELEMENTS BY NEUTRON IRRADIATION, by Engelbert Broda. 152 pages, tables and diagrams, 14 × 22 cm. New York, Cambridge University Press, 1950. Price, $2.75.

One of the Cambridge Monographs on Physics, this little book presents a surprising wealth of facts concerning the production and properties of radioactive isotopes. By stating only the heart of each investigation or phenomenon along with its reference, and omitting the usual extraneous commentary, the author has provided a very useful primary reference source for the phases covered, which deal mainly with work done since 1936. In fact, there are approximately a thousand chapter-end references, which is considerable for any book, let alone one of this size. At times, however, the frequency of names in the text almost results in a treasure hunt on the part of the reader, for the text words themselves.

The application of radioisotopes as tracers is not considered in this book, nor is the effect of radiation on molecules (the latter field is properly known as radiation chemistry). The principal subjects covered include, however, the basic considerations involved in the use of radiotracers, such as solubility distributions, carriers, and adsorption phenomena.

The production of radioelements by nuclear synthesis is well outlined and referenced, especially with respect to pile methods. This is logically followed by a brief chapter "The Chemical Aspects of Nuclear Fission." An interesting section of the book discusses the new radioelements, such as the transuranic, and their chemical properties. The remainder of the book is devoted to specific radiochemical effects such as the Szilard-Chalmers, and to a summary of techniques involved in making radiochemical measurements.

There are a few statements that seem dogmatic, such as "to a good approximation all isotopes of an element show identical chemical behavior." Recent research has shown that isotopic kinetic effects are by no means negligible. However, this in no way impairs the value of the book as a pocket guide to those aspects of radiochemistry therein covered.

A. L. MYERSON

THE STORY OF METALS, by John W. W. Sullivan. 290 pages, illustrations, 13 × 20 cm. Cleveland, American Society for Metals and The Iowa State College Press, 1951. Price, $3.00.

The history of metals from the earliest times up to the present is traced in this small volume. It also describes in a non-technical manner many important aspects of modern metal technology.

The book consists of ten chapters: the first deals briefly with the origin of metals and the composition of the earth's crust; the next four chapters are devoted to the history of metallurgy; and the last five chapters contain descriptions of modern metallurgical processes and techniques. Nearly all the metals and many of their commercially important alloys are discussed.

The book is printed on an excellent grade of paper and contains numerous illustrations. There is a 27 page glossary of technical terms at the end of the book. Since the subject matter is treated from a very elementary point of view, this book should be of most interest to non-technical persons interested in learning more of the science of metallurgy.

M. D. EARLE

AIRPLANE AERODYNAMICS, by Daniel O. Dommasch, Sydney S. Sherby and Thomas F. Connolly. 520 pages, illustrations, 16 × 25 cm. New York, Pitman Publishing Co., 1951. Price, $6.50.

The authors have designed this book for students having a background in college physics, calculus, mechanics and dynamics. Although the text was originally intended to reveal the methods of aeronautical engineering to pilots and test pilots who were not already aeronautical engineers, the emphasis throughout the book on the fusion of design theory with the operational problems of the pilot makes it an excellent textbook for aeronautical engineering students.

The properties of the atmosphere are presented in an introductory chapter, followed by one on the dynamics and thermodynamics of air. The Bernoulli equation is derived and applied to the problems of air speed measurement and the temperature rise at the surface of bodies in supersonic flight.

A third chapter discusses the types of fluid flow, stressing the use of Reynolds number criteria. Vortex flow and the Kutta-Joukowsky theorem are presented. The composition of plane flows follows intuitive lines.

Airfoil theory is given in an elementary manner; however, the important dependence of the lift, drag, and moment coefficients on Mach number and Reynolds number is carefully pointed out.

A section on wing theory covers the important wing parameters. The use of high force-divergence Mach number as a design criterion for high subsonic speed planes is emphasized. The operation of the propellor is interpreted with the momentum theory, the blade-element theory, and the vortex theory. Propellor selection is illustrated. The principal types of power plants—reciprocating engine, gas turbine, ramjet, pulsejet, and rocket—are discussed

briefly. The problems of performance and maneuvers, longitudinal stability and control, and lateral stability and control are treated thoroughly. A final chapter discusses the design and operational problems of seaplanes.

The illustrative examples covering important design and testing problems should be quite helpful to designers as well as test pilots. The theory and problems require no mathematical background beyond integral calculus.

HOWARD H. BROWN

METHODS OF OPERATIONS RESEARCH, by Philip M. Morse and George E. Kimball. First edition, revised, 158 pages, diagrams, 20 × 28 cm. New York, John Wiley & Sons, Inc., 1950. Price, $4.00.

During the last war first the British armed forces and then our own military establishment became aware of the great potency of the operations research approach to certain problems. As defined by Morse and Kimball, "Operations research is a scientific method of providing executive departments with a quantitative basis for decision regarding the operations under their control." Doubtless systems engineering and similar types of problem analysis have much in common with operations research, but operations research differs in that its problems are of such wide scope and its sources of techniques and models for the solution of these problems are drawn from many and diverse fields of study. Thus it would not be unreasonable to expect to find an operations research team consisting of a physicist, a psychologist, and perhaps an electrical engineer, each member contributing both his stock of information as well as the frame of reference peculiar to his own technical discipline. The results of the analysis, observations, and possible experimentation conducted by such a team would be in the form of a solution of the problem which would define the parameters of the problem at hand and their interrelations insofar as these could be found. This information, together with whatever other factors the executive believed pertinent, would enable the executive to make his decision. Operations research is an aid to, not an infringement on, the executive function.

Probability and statistics are extremely important mathematical tools of operations research and the authors of this book provide an excellent introductory chapter on the subject so as to enable the reader to understand the subsequent chapters. Suffice it to say that the following chapters describe various military problems and typical solutions. Although it should be stressed that the implications of the methods transcend military applications alone. The solutions exemplify the methods of operations research and point up some of the characteristic questions. For example: what shall be chosen as the criteria of effectiveness for a given operation, such as anti-aircraft fire or anti-U-boat warfare? The volume ends with a detailed discussion of how to set up an operations research group. This book should be of great value to persons interested in the future possibilities for industrial management, to persons interested in how the military copes with the enormous complexities of modern warfare, and to all persons interested in learning how to apply new methods to the prediction and the modification of results of complex operations. All in all, the declassification and publication of this volume perform a great service in making available detailed information on a new, and hitherto relatively unknown, branch of applied science.

EZRA KRENDEL

AN OUTLINE OF SCIENTIFIC CRIMINOLOGY, by Nigel Morland. 287 pages, plates, 13 × 19 cm. New York, Philosophical Library, 1950. No price.

This book of 284 pages is an extensive, but superficial outline of the use of scientific techniques in the investigation of crimes and the prosecution of criminals. The author deals with the more romantic aspects of fingerprinting, identification, ballistics, forensic chemistry, etc., and cites several dramatic examples of their historical importance. The center 12 pages of the book contain photographs of forensic science at work, photographs such as would appear in "B" picture montage. However, the bibliography is moderately complete, and the additional notes at the end of each chapter contain interesting and amusing information. The book is recommended as light reading for persons with a casual interest in criminology.

M. HALBERT

SUPERFLUIDS, by Fritz London. Volume 1, Macroscopic Theory of Superconductivity. 161 pages, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.00.

The development of a theory of superfluids or quantum liquids is based upon two interesting physical phenomena, both of which apparently allow a flow of matter without friction. The first is the ability of liquid helium when cooled below 2.19° K. to pass through extremely fine capillaries or fine cracks which would be practically impossible for any ordinary liquid. The second is represented by the superconducting state of electrons as exhibited by a metallic superconductor, where at very low temperatures electric charge is transmitted without any measurable voltage difference between terminals. These experimental observations the author attempts to explain by assuming them to be macroscopic evidences of quantum mechanical mechanisms. The proposed theory is based on an attempt to explain the Meissner effect that superconductors are not only ideal conductors ($\dot{B} = 0$) but also ideal diamagnetics ($B = 0$). If this be so, then a new relationship to replace Ohm's law must exist between the electric current and the magnetic field. This relationship suggests the quantum mechanical interpretation to describe the state of those electrons which become superconducting as a state of long range order with respect to the local mean values of the momentum vector.

In the first of five chapters the author discusses the properties of superconductors and some of their thermodynamic relationships. This is followed by mathematical discussions of: the electrodynamics of superconductors in terms of special solutions of Maxwell's equations with specified boundary conditions; perfect diamagnetism; non-viscous flow and superconductivity; uniqueness theorems; rotating superconductors; and microwaves in superconductors.

Certain materials exhibit a microstructure consisting of a series of alternating normal and superconducting domains. A theory for such a mixture of domains would be extremely complex and difficult of experimental verification. The author, however, is undaunted, and proposes a theory of mean values of field strength which can be checked experimentally.

The remaining two chapters are devoted to: phase transitions of thin films; surface energies; and a molecular theory of superconductivity.

A theory to explain superfluids must at the present time be a flexible one, because experimental evidence is still being collected and because of the complexity of the problem. Physicists, physical chemists, and engineers interested in low-temperature physics will find much of interest in the volume for these reasons.

S. CHARP

THE NOMOGRAM: THE THEORY AND PRACTICAL CONSTRUCTION OF COMPUTATION CHARTS, by H. J. Allcock and J. Reginald Jones. Fourth edition, revised by J. G. L. Michel, 238 pages, diagrams, 14 × 22 cm. New York, Pitman Publishing Co., 1950. Price, $3.75.

This book is primarily concerned with nomograms of the third class, although some attention is given to nomograms of the fourth and higher classes. The authors discuss the various types in each class including practical construction methods. In the case of nomograms of the third class, discussion is concerned with those of Genus Zero, I, II, and III. Multiple grid and alignment techniques are discussed for the higher classes. A noteworthy feature is the summary in Appendix I which gives the standard forms (of nomograms) of the third class.

A. D. H.

INTRODUCTION TO SERVOMECHANISMS, by A. Porter. 154 pages, illustrations, 11 × 17 cm. London, Methuen & Co., distributed in U.S. by John Wiley & Sons, Inc., 1950. Price, $1.75.

This book is the latest of the Methuen Monographs series and is written for the average scientific worker rather than those engaged in work in the field of servomechanisms. It is easily understandable, concise and well presented. The classical approach is used throughout in order to avoid mathematical incumbrance for those unfamiliar with the methods of the Laplace transformation.

L. SCHIEBER

BOOK NOTES

PLANT LAYOUT, by Randolph Mallick and Armand T. Gaudreau. 391 pages, illustrations, 16 × 25 cm. New York, John Wiley & Sons, Inc., 1951. Price, $7.50.

Plant Layout is a practical book covering engineering techniques for laying out new plants and modernizing existing ones. The authors have presented all phases of plant operation, planning and designing, material handling, equipment and development of manufacturing techniques, and have included useful information on capital outlays, operation costs and returns on investment. The complete flow sheets, floor plans, tables and illustrations will serve as practical guides to both management and plant engineers.

ALTERNATING CURRENT FUNDAMENTALS, by Joseph J. DeFrance. 216 pages, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1951. Price, $3.50.

This text is the second of a series of books written for the purpose of training electronic technicians. The author has maintained his purpose of presenting alternating current theory and its application to electronics to students of electronics in a readable style, in a clear manner and with diagrams that are illustrative of basic concepts. The material covered includes sine waves, complex waves; resistance, capacitance and inductance as a circuit element; series and parallel circuits, and vector algebra for a-c. circuits.

INTRODUCTORY QUANTITATIVE ANALYSIS, by Ernest H. Swift. 534 pages, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1950. Price, $3.90.

This book was written to provide text material for a one semester course in quantitative analysis, and it includes procedures serially numbered in each chapter and an appendix of questions, problems and demonstration experiments. Material has been organized to familiarize the student with precise techniques and physical measurements characteristic of quantitative chemical methods, with principles of the methods and mass action principles to show factors controlling reactions. An excellent book for students working alone or persons wishing to review quantitative techniques.

QUANTITATIVE ANALYSIS: A THEORETICAL APPROACH, by William Rieman, Jacob D. Neuss, and Barnet Naiman. Third edition, 523 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., 1951. Price, $5.00.

This third edition of a well known text deals almost entirely with the analysis of inorganic substances. The laboratory procedures have been revised. Changes include the new Kjeldahl procedure which eliminates distillation, application of electroanalysis, use of formal redox potentials and a chapter has been added on the theory and use of ion exchange. The chapter on colorimetry contains theory and use of photoelectric instruments of both one and two cell types. The book is of value and has been brought up-to-date on quantitative procedures.

Portable Television Camera.—A new portable television camera and transmitting station, designed to operate in the field as a one-man back-pack unit, was demonstrated by L. E. Flory, of the RCA Laboratories, at a meeting of the Institute of Radio Engineers in The Waldorf-Astoria Hotel.

Weighing only 53 lb., the back-pack station is planned to function with its own battery-power supply. It has a range of approximately one mile. Because of its easy portability, numerous applications for the new equipment are foreseen by RCA research engineers. Among these are news coverage, with television-equipped reporters flashing pictures and commentary directly to editorial rooms, and remote industrial viewing and control.

The new transmitter operates in conjunction with a control station which may be located as far as a mile from the camera. Signals corresponding to the scene being televised are transmitted to the control point on an ultra-high frequency with a power of two watts. In addition to acting as a monitor for the televised picture, the control point performs two other functions. It sends out a stream of pulses which stabilize the camera and can be used also to issue vocal instructions to the cameraman.

Recent developments in the design of pencil-sized tubes and other sub-miniature component parts made possible the impressive reduction in bulk and weight of the equipment.

The back-pack is carried in knapsack fashion, suspended from the narrator's shoulders by flexible straps. Two small antennas extend from the top of the pack and are used respectively to transmit the picture signal to a base station and to receive voice and control signals from that same point.

The camera is an adaptation of the RCA industrial TV camera using the Vidicon tube. As an added feature, the camera includes a miniature kinescope picture tube which serves as a view-finder for the cameraman. Through it he is able to see an exact reproduction of the scene on which the camera lens is focused.

The equipment contains 42 tubes which, with their associated circuits, provide all synchronizing frequencies for a standard 525-line, 30-frame interlaced television picture. Included in the unit are the battery-operated power supply, deflecting circuits, amplifiers, and a radio receiver for receiving instruction and other essential information from the control point. A single battery operates the portable station for about 1½ hr.

The narrator-cameraman's voice is picked up and transmitted through the combination of a small microphone built into the camera case and an ingenious electronic circuit which adds the voice signals to the picture signals as they are radiated to the control point.

Research and development of the portable television equipment were carried out by Mr. Flory, W. S. Pike, Jr., J. E. Dilley, and J. M. Morgan, of the RCA Laboratories, under the direction of Dr. V. K. Zworykin, Vice President and Technical Consultant.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE MCDONALD, DIRECTOR

IONS AND RADICALS IN BIOCHEMICAL PROCESSES*

BY

WILLIAM A. MOSHER

One of the schools I attended was an institution where free radicals held sway and anyone who mentioned the work "ion," at least in the organic sense, was promptly thrown out. I had the bravery once to mention ions, so I left that school and went to a university where ions held sway and where anyone mentioning the term "free radical" was immediately branded one and dismissed. After these rather trying experiences I have spent as much of my thoughtful and experimental time as possible exploring both of these fields to try to find the common ground where ions begin and radicals end; to determine, if possible, those reactions in which ions rule and those in which radicals are involved.

Some of the things I will talk about will be based on complete speculation and many of the topics will have little apparent relation to biological antioxidants, although the connection is real. First, some rather general background material; let us consider the perfectly generalized reaction

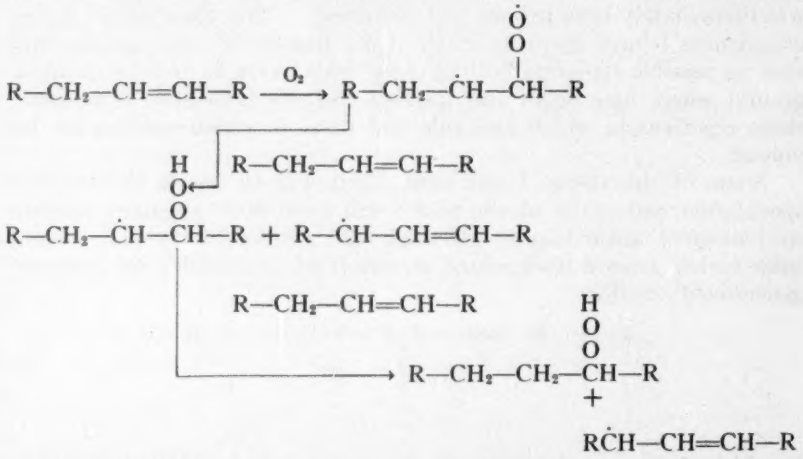
$$A^+ + :B^- \rightleftharpoons A:B \rightleftharpoons A:^- + B^+$$
$$\Big\Vert$$
$$A\cdot + \cdot B$$

in which $A:B$ represents any organic compounds in which two groups A and B are joined together by a covalent bond, a pair of electrons. There are only three ways in which this bond may be ruptured or formed and these ways are indicated. If the breakdown is into A^+ and $B:^-$ then group A becomes deficient in electrons while B now has an excess. If the respective charges are associated with carbon atoms the positive ion is referred to as a "carbonium ion" and the negative ion as a "carbanion" (1). This splitting of a molecule into ions of unlike charges is referred to by British workers as a "heterolytic" split. If the rupture is such that one electron of the binding pair goes with each of the two groups then neutral particles, in this case free radicals, are formed and the split is called "homolytic." The ions

* Reprinted from *Biological Antioxidants*, Transactions of the Fourth Conference, Josiah Macy, Jr. Foundation, December 8–9, 1949, New York, N. Y.

have charges which will cause attractions and repulsions to occur; the radicals have no charges but do possess magnetic moments. Electrons in atoms are moving charges and as such they produce magnetic fields. Electrons spinning in opposite directions pair and in this way balance the magnetic fields. Molecules, atoms, or atomic groups containing only paired electrons show no magnetic properties while those with odd electrons, as is the case with the free radicals, do show magnetic properties. Just as the desire for neutralization of charges causes reaction between ions, so does the desire for cancellation of magnetic moments bring about the union of radicals. This "desire" might better be called the second law of thermodynamics.

These breakdown processes are very seldom spontaneous but generally require some outside intervention. The separation of charges or of magnetic fields requires work, whereas the reverse process, their



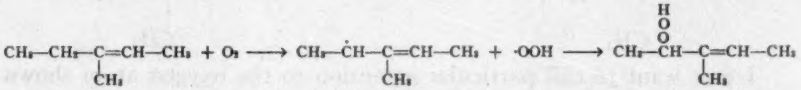
union, liberates energy. I am sure that either homolytic or heterolytic breakdown is possible in the same molecule depending on the circumstances. Evans in England has looked into the matter from the energetics and has shown that the homolytic split, the radical dissociation, requires less expenditure of energy than the ionic split (2). He has added parenthetically, however, that the situation may be completely reversed when working in solvents of high dielectric constant where the energy of solvation of ions may be high, even higher than the energy required for separation of the charges. In the highly polar media of biological systems the energetics will permit either type of breakdown.

Shown below are some of the ways in which free radicals may be generated by attack of oxygen on unsaturated systems. I don't believe Farmer (3), from whose work this is taken, has proved that

there is appreciable attack of oxygen on the double bond itself but has offered this as a suggestion as to how the initiating radicals might be formed. Perhaps one molecule in 100,000,000 reacts this way. If such does occur we form a diradical with an unpaired electron on the oxygen and one on the carbon and the chain reaction can get under way.

In the course of its vibrations and thermal motions, the free radical position on oxygen will seek some means of balancing its magnetic deficit. Were it to remove an H· from a carbon atom, and it generally does this at a methylene group activated by a double bond α to it, the result would be the formation of a hydroperoxide group. Such hydroperoxides are certainly the first isolable products in the oxidation of unsaturated compounds with molecular oxygen. We still haven't taken care of the lone electron left on the carbon atom. It could get an H· in the same manner, add to a double bond and start a polymerization, or several other possibilities.

There is also no reason why oxygen itself, a rather aggressive reagent at times, might not go in and remove an activated hydrogen directly as is indicated below. Although either addition of oxygen to the



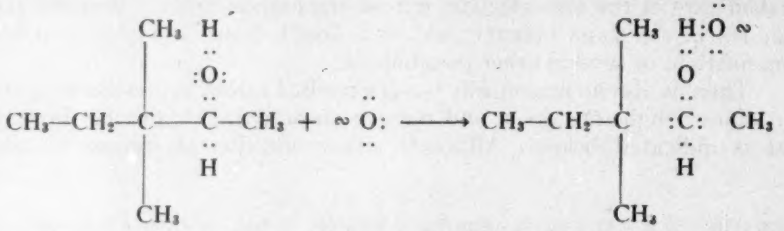
double bond or direct attack on methylene hydrogen is logical enough, I personally prefer the second. The double bond, with its very high electron density, is not very susceptible to attack by a molecule with such a high electron density as oxygen; repulsion would be expected rather than attraction. The electronic forces at play would certainly favor α-methylene attack. There is need for more work here; labeling the carbon atoms, either with radiocarbon or with methyl groups in place of some of the hydrogen atoms, could determine the initial attack mechanism.

You will recognize the mechanism of oxidation here as a "one electron process;" the electrons are involved one at a time. In many electrochemical reactions the evidence is clear for such mechanisms. Dr. Michaelis had a great deal to do with establishing this sort of process in the organic field. The semiquinones and other compounds of that type fit nicely into the picture (4). On the other hand, there are some rather simple oxidations which are difficult to explain on a one electron or free radical basis.

I am going to pass now to the historical development of some new ideas in oxidation of organic substances. This is an example of how organic reaction mechanisms come into being, and the work may raise some suggestions and questions more closely related to our specific biochemical interests.

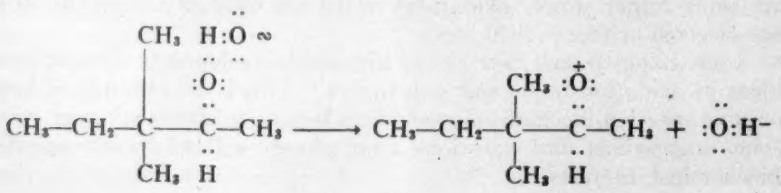
In carrying out a perfectly straightforward organic reaction, the oxidation of methyl-tertiary-amylcarbinol with chromic anhydride to the corresponding ketone, small amounts of an unexpected product, tertiary-amyl alcohol, were found. Other highly branched alcohols behaved in the same manner (5). We are now certain that this type of cleavage reaction is involved in practically all alcohol oxidations (6). In some cases only fractions of a per cent of split products will be formed while in others the yields will approach 100 per cent.

If we work in the aromatic field with alcohols containing the triphenylmethyl group we would expect to find triphenylmethyl peroxide in the presence of oxygen, were free radicals involved. This peroxide, or compounds which might logically be derived from it, are not found. We have therefore tried to explain the facts on an ionic basis as follows:



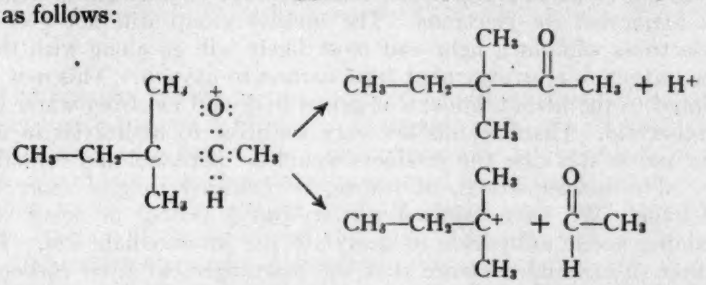
I first want to call particular attention to the oxygen atom shown with only six electrons. I do not want to give the impression that any such species exists in an oxidation reaction, especially one involving chromic anhydride, potassium permanganate, or similar reagents. There is certainly something else hooked onto this oxygen, probably chromium with other oxygen atoms. From the standpoint of the reaction and its products, however, we can simplify the process for discussion by representing the oxygen in this manner. The suspicion is that coordination compounds between alcohol and oxidizing agent fit into the picture—we are getting evidence of this in experiments now in progress. The first step is probably a coordination of oxygen in the oxidizing agent with the hydrogen of the hydroxyl, which is certainly the most polar hydrogen available in the system. Coordination might be between chromium of the oxidizing agent and the oxygen of the hydroxyl group, however.

The following might happen when the coordination complex breaks down:



A hydroxyl ion, of great intrinsic thermodynamic stability, is split out and an intermediate containing an oxygen atom with only six electrons is formed. This intermediate ion will have only a transitory existence. Carbonium ions, with a positive charge on the carbon, have only the briefest existence. Something as electronegative as oxygen, something with the pull for electrons which oxygen has, would not, with a positive charge, have anywhere near as long a life as a carbonium ion. At this point you may well say, "I can't imagine anything stealing electrons from oxygen." Remember, oxidation is not the simple thing energetically that, say, acid-base neutralization is; oxidation requires potential. The energy involved is much higher than in simple addition reactions. The potential of an oxidation reaction may very well be the energy required to steal electrons from a combined oxygen atom. But getting back to the mechanism, how can we get this transitory intermediate into stable end products?

The stabilization routes suggested to explain the observed products are as follows:



Loss of the hydrogen nucleus from the carbon bearing the deficient oxygen atom would produce the expected ketone; rearrangement of the pair of electrons binding the tertiary-amyl group would split the molecule and yield acetaldehyde, which has been found, and the tertiary-amyl carbonium ion. This ion would form alcohol and hydrogen ion in the presence of water:

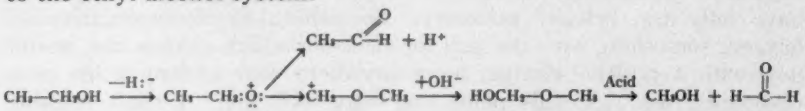
$$R^+ + HOH = ROH + H^+$$

In highly polar media the loss of these groups is facilitated by dipole action working in concert with the tendency on the part of the intermediate to stabilize through expulsion.

When ethyl alcohol is oxidized with reagents we always think in terms of the usual products, acetaldehyde and acetic acid. The surprising fact is that the oxidation of ethyl alcohol with chromic acid, manganese dioxide, permanganate, vapor phase dehydrogenation, or bacteria yields in all cases very small amounts of formaldehyde. How is the formaldehyde formed? Organic chemists do not like to think of a reagent getting in and breaking a bond in a saturated molecule

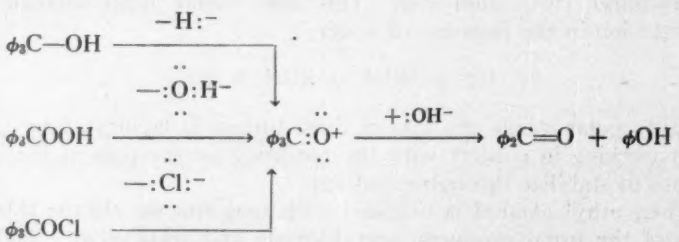
at ordinary temperatures. At 400° you can crack molecules but not usually at 25° in water solution.

The following shows the application of the processes described above to the ethyl alcohol system.



First, a hydrogen atom with two electrons, a hydride ion, is removed from the hydroxyl group to give the deficient-oxygen-containing intermediate. Acetaldehyde is formed if a proton is expelled in the stabilization process but if the pair of electrons is attracted from the β position by the oxygen the situation may follow a different course than in the case of the highly-branched alcohols, although the end result is the same. In the present case we have a methyl group attached to the electron pair, a small group rather than the large tertiary group with its weak attraction for electrons. The methyl group will not give up the electrons without a fight and most likely will go along with them to the oxygen, a rearrangement from carbon to oxygen. This new ion is related to the hemi-acetals; if it gets a hydroxyl ion from water it is a hemi-acetal. Hemi-acetals are very sensitive to hydrolysis in acid media and in this case the products would be methanol and formaldehyde. The former would, of course, be oxidized to give more formaldehyde. We have oxidized tertiary-butyl alcohol in acetic acid containing acetic anhydride to acetylate the intermediate ion. This is rather substantial evidence that the rearrangement from carbon to oxygen does occur and this fact points to a center of deficiency on the oxygen atom.

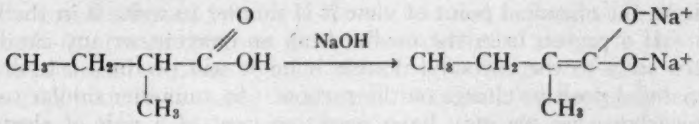
Below, there are assembled three distinct reactions all involving the triphenylmethyl system and all giving the same products but with different reagents (7).



A common reaction intermediate is strongly indicated. Phenol itself is not isolated from the chromic anhydride oxidation but products derived from it by further oxidation are. If oxidation removes $H:^-$ from the carbinol, acid removes OH^- from hydroperoxide, and Ag^+ removes Cl^- from hypochlorite a common intermediate with a deficient

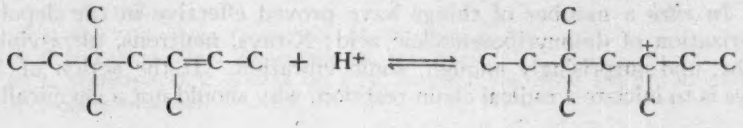
oxygen atom would be formed. The decomposition of this intermediate would give phenol and benzophenone. It is interesting to note that the Hercules Powder Co. has very recently developed a commercially feasible process for manufacturing phenol through the acid-catalyzed decomposition of tertiary hydroperoxides containing a benzene ring attached to the tertiary carbon atom.

Now let us dig a bit deeper into some biochemical processes. The matter of β-oxidation as originally developed by Knoop (8) and extended by Dakin (9) has been the subject of much confusion. More recently Schoenheimer and Rittenberg (10) have confirmed the two-carbon oxidation using tagged-atom techniques. An acceptable mechanism has not yet been presented. The following offers a possible explanation consistent with the biological facts as well as with some new chemical evidence presented below.

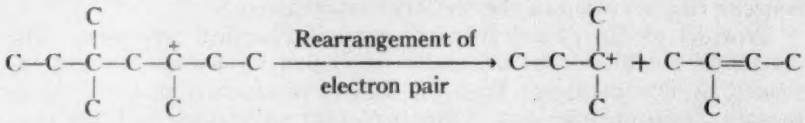


The organic chemist is convinced of α-activation but β-activation is very hard to accept. Oxidation of organic acids in acid media leads to α-hydroxy or α-keto acids. Dakin was able to reproduce *in vitro* some aspects of the β-oxidation observed biologically but only in alkaline media. This alkaline medium might very well convert at least a fraction of the acid to an enol salt. Our data indicate that the rate of β-oxidation *in vitro* is essentially the same as the rate of base-catalyzed racemization of acids with an optically active α carbon atom. In basic media we would expect the loss of true unsaturated nature in the carboxylate ion to minimize α-activation; on the other hand, the tendency toward a carbon-to-carbon double bond in the enolate should give reasonable activation in the position β to the original carboxyl group. Chemically speaking we may look upon the β-oxidation of Knoop as an α-oxidation process.

The breakdown of phosphorylated hexose sugars into triose sugars is one of the most important of all reactions taking place in biological systems. To find an analogy let us go back to some simple aliphatic hydrocarbon chemistry. On treatment with strongly acidic reagents, 3,5,5-trimethyl-2-heptene breaks down quantitatively into trimethylethylene (11). We can represent the mechanism as first involving the addition of a proton to the double bond to form a carbonium ion. For simplicity, I am showing only the carbon skeletons:



The carbonium ion may stabilize itself in a number of ways but the method it selects under these conditions is the rearrangement of a pair of electrons from the position β to the positive charge, just as was the case in the oxidations. Because the group involved is large, only the electrons rearrange and the molecule is broken:



the tertiary-amyl carbonium ion would lose a proton readily and give a second molecule of trimethylethylene.

The same reasoning may be carried over to fructose-1,6-diphosphate. It may or may not be necessary for the oxide ring in fructose to open but from the chemical point of view it is simpler to write it in the keto form. If a proton from the media, from an enzyme, or any catalytic surface adds to the carbonyl double bond it will produce a hydroxyl group and a positive charge on the carbon. In a manner similar to our hydrocarbon case we may have rearrangement of a pair of electrons to break the molecule of sugar in the middle to produce, by simple steps, glyceric aldehyde phosphate and dihydroxyacetone phosphate. Here is a consistent, ionic explanation of dealdolization. I want to discuss one more topic and that has to do with the effects of irradiation and other agents on nucleic acids (12). We are all aware of the great amount of research on nucleic acids throughout the world because of the apparent role of these compounds in genetics and cancer problems. We were stimulated by some work of Greenstein's at the National Institute of Health in which desoxyribosenucleic acid was irradiated *in vitro* with X-rays (13). The molecular weight of the acid goes way down in the process. The most exciting thing to us was the fact that the reduction of viscosity continues for a time after radiation has been stopped. It looked to us as though a chain reaction of some kind was involved.

Working in water solution there are a number of things which might happen. Weiss (14) showed that X-rays might produce hydroxyl radicals and hydrogen atoms from water. The hydrogen atoms could react with dissolved oxygen to produce more hydroxyl radicals and so on. If thiourea or some other SH containing compound is present you can, as is well known, break up such free radical chains. Substances of this sort are used to control the length of chain reactions in commercial synthetic rubber polymerizations.

In vitro a number of things have proved effective in the depolymerization of desoxyribosenucleic acid: X-rays, neutrons, ultra-violet light, and surprisingly enough, sonic vibration. If the action of X-rays is to initiate a radical chain reaction, why should not a chemically-

induced radical chain work just as well? It does. Ferrous ion and ascorbic acid, a good chain-starting combination, will give the same reduction in viscosity, molecular weight (by sedimentation), and streaming birefringence as obtained with X-rays. Ascorbic acid works well alone but it is even better with hydrogen peroxide. If we irradiate dry nucleic acid, or a frozen solution there is no depolymerization. Water plus the chain-initiating mechanism must be present. The effect in a solution of ethylene glycol, which also can yield hydroxyl radicals on irradiation, is clearly noticeable but is not as great as in water solution.

If you X-ray rats, remove the thymus, and isolate the nucleic acid, the molecular weight is greatly reduced when compared with the controls. As is generally known, the size of the thymus goes way down. When rats are irradiated with 1000 r and sacrificed after varying times, the effect on the animal appears to be even longer delayed than is the case in the test tube. If the nucleic acid is obtained immediately after X-ray treatment there is a marked change in purine to pyrimidine ratio but not too much change in molecular weight. As the rat lives the depolymerization continues and is essentially complete after 24 hours.

If chain-stopping agents, thiourea for example, are added to *in vitro* samples of nucleic acid there is very marked protection from the destructive effect of X-rays, chemically-produced radicals, and other physical agents. Because we can duplicate the effect of X-rays with chemical agents of known mechanism and because we can inhibit, *in vitro*, both types with the same agents, we have confidence in a common mechanism, and that a radical mechanism.

Next we tried *in vivo* experiments. Rats fed or injected with thiourea showed only about half the mortality from X-rays as did the control animals. The desoxyribosenucleic acid in the thymus of the X-rayed rats protected with thiourea was of a much higher degree of polymerization than the controls. We can look upon this entire matter of protection as a type of antioxidation although it looks quite different in its outward aspects from reactions involving oxygen. We suspect that oxygen tension is a factor in the radiation process.

We have tried to bring together some oxidation type reactions which are clearly radical in nature and some that are most certainly ionic in nature. In our future considerations of biological antioxidants I believe we must keep both mechanisms before us.

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CURRENT TOPICS

Fire-Control System for Anti-aircraft Artillery.—Development of a new and far more effective fire-control system for automatically aiming anti-aircraft artillery was announced recently by Bell Telephone Laboratories, with the approval of the Army Ordnance Corps for whom the project was undertaken.

The new fire-control system—a virtual maze of wires, resistances, switches and vacuum tubes—is already in production by the Western Electric Company, manufacturing and supply unit of the Bell System, also operating under an Army Ordnance contract.

The new system is an outgrowth—with many radical improvements—of the famous Bell Laboratories electrical gun director and its associated radar systems, which proved so remarkably effective against Nazi planes and "buzz bombs" in World War II. This earlier fire-control system, a major "secret weapon" of the war, worked almost entirely automatically. Radar found and "tracked" a hostile plane, and fed continuous information concerning its location into a computer, or "electrical thinking machine," which was the heart of the system. At the same time, data relating to wind velocity, muzzle velocity of the shells, temperature and similar factors, were given to the computer. This machine then automatically calculated where a shell should explode to bring the plane down, and automatically aimed the guns to do just that.

The new fire-control system, adaptable to firing either 90 mm. or 120 mm. anti-aircraft batteries, operates on the same general principle as its predecessor, but many improvements and refinements have been incorporated. As a result, it is far more effective and flexible than the World War II version.

Details of the new system are classified for security reasons but it can be said that much more information concerning hostile craft can be assembled and processed, and the entire system works more accurately, more quickly and more easily. This is due to a number of new circuits and circuit elements which have been introduced into the system. Naturally the system is geared to handle high-altitude, high-speed targets, and to take into account advances in plane and projectile performance which have been made since the war.

The new fire-control system is extremely compact and portable, and can easily and quickly be moved cross-country anywhere the gun battery can go. In addition, the equipment is unusually light in weight and can readily be transported by plane. It is also floatable so that it can be towed across rivers or lakes.

The housing for the complex electronic equipment was designed by the Douglas Aircraft Company, Inc., and the housing units are being manufactured by The Glenn L. Martin Company, both as subcontractors.

The Army Ordnance Corps asked the Bell Laboratories–Western Electric team to undertake research, development work and manufacture of the fire-control system not only because of their wide experience with the highly successful gun director and radar of World War II, but also because of their unique qualifications in the field of electronics. Many of the basic principles

of fire-control systems, for example, are similar to those employed in the complex switching mechanisms which underlie the dial system of the modern telephone network. Many of the components of the intricate computer which is the heart of the fire-control system—switches, relays, vacuum tubes and wave guides—as well as some of the circuits, are also found in the telephone system. The close association between the Laboratories and Western Electric in the Bell System for many years also provides an ideal working arrangement for ensuring complete coordination between research and development on the one hand and manufacturing on the other, in putting these new and highly complex fire-control systems in production.

Television in Service of Science and Industry.—An electronic instrument, in the form of a television microscope, displayed at the Institute of Radio Engineers Convention in the Grand Central Palace, attracted widespread attention through its revelation of assorted microscopic bugs heretofore unseen by the eye. This new instrument is a product of research at RCA Laboratories, Princeton, N. J.

Visitors at the exhibition saw demonstrations of this remarkable device which operates as follows: A small, special-model television camera about the size of a home movie camera is mounted over the eyepiece of an ordinary microscope. A drop of stagnant water, full of microscopic life, is put under the lens—just as in a high-school biology class. But instead of looking at the bugs through the microscope, you see them on the television receiver screen. The picture is transmitted over a small coaxial cable from the camera to a compact receiver-control unit about the size of a small table-model television set. It is especially adaptable to classroom instruction.

Rigged to another television microscope is an electronic particle counter. This device allows a laboratory technician to obtain an instantaneous count of the number of tiny particles appearing, for example, in one square millimeter of a specimen. Heretofore, counting particles in a specimen has been an eye-straining job requiring the laboratory assistant to peer through the microscope and tick off the particles with a hand counter.

In another setup, two of the special television cameras are mounted over a binocular microscope. The pickup tube in one camera is sensitive to red light; the pickup tube in the other is sensitive to blue light. Two receiver units are placed side by side. In this way, substances in a specimen which are more visible under blue light can be seen on one screen; ones which are more visible under red light can be seen on the other.

Servo Amplifier Available for Industrial Use.—A compact, easy-to-use, and easy-to-maintain servo amplifier has been placed in production for industrial use by Servomechanisms, Inc., Mineola, N. Y. Designated the SA-203, it is the first of a series of 60-cycle amplifiers which Servomechanisms has developed to meet the exacting requirements of spatial adaptability, simple training, and instant maintainability.

Like all Servomechanisms' products, the SA-203 has been designed as a small plug-in unit, in which all the electronic elements required for one function in the control system are packaged together. Additional plug-in facilities within the unit, comprising the transmission control networks, permit the

adaptation of the amplifier to widely varying requirements for gain, phase shift and damping. Therefore, it may be used in many different applications, with a common stock of spares required for all uses.

Functional packaging of the amplifier elements simplifies maintenance since a unit which gets out of order can be easily unplugged and replaced with another. Similarly, it simplifies training since the unit is small and easy to understand, containing only the specialized components for one function.

With the elements packaged compactly in the smallest possible volume of space, the SA-203 measures only 5 in. in width by 8 in. in length by 4¾ in. in height, and weighs only 5½ lb. This makes it easy to plug into a chassis with other servo units or to install in a small volume where space is at a premium.

The SA-203 has been proven in use on Government equipment during the past year, and similar functionally-packaged amplifiers and electronic components of the 400-cycle type have been used by the military for the past several years. Now released for industrial use, it is available for application in such equipments as analogue computing devices, laboratory measuring instruments, and industrial process controls. Being a standard item, it is expected to be particularly useful to manufacturers of military equipment in developing control systems for accurate and automatic processing and inspection, which are essential to quantity production. It is also suitable for laboratory use in establishing the requirements for new control systems.

Twin Stars.—New observations of twin or variable stars by University of Wisconsin astronomers lend support to the theory held by many present-day scientists that the universe is slowly evolving toward a stage of equilibrium in which all stars will be about the same size and brightness.

Twin stars are located very closely together—so closely that they appear as a single point of light even through the most powerful telescope. They are called variable because cyclic variations in the intensity of light coming from them first caught the attention of astronomers. This variation is the result of eclipses as the twins revolve around one another, causing a decline in the amount of light received in astronomers' telescopes.

While measuring the light coming from twins known as "U Cephei," University of Wisconsin Prof. C. M. Huffer recently found that the hotter and smaller of the twins was brighter after eclipse than it was before. Not only that, he found that the intensity varied from night to night even though it was measured at exactly the same stages of eclipse.

This variation leads Professor Huffer to believe that streamers of gas are spinning away from the small, hot star—like sparks from a pinwheel—and are being gathered up by the larger twin through gravitational attraction. Just before eclipse, these streamers partially obscure the light from the small twin, causing variations in the readings made before and after eclipse—and night to night—with the UW Washburn observatory telescope and photoelectric cell.

The twin stars of "U-Cephei" revolve around one another every two and one-half days. The gas streamers apparently come away in huge arcs from the hot twin and gather most densely in the region between the stars. Because

of the centrifugal force caused by the rotation of the hot twin on its axis, the gas is thickest toward one side—the side through which light to the earth passes during pre-eclipse stages.

Professor Huffer's observations have led him to believe there is always an exchange of material from the hot to the cold twin in all of the twin stars, wherever there is a difference of temperature between the twins.

One of every four or five stars in the universe is a twin star, Professor Huffer points out. "This means there must be a lot of energy and matter being transferred by this means throughout the universe," he says. He also adds that the UW astronomers would like to make a thorough survey of stars of this type.

The UW astronomers have been working on "U Cephei," among other variables, for five years. The gas streamer effect, however, was noticed only recently.

Horizontal Smokestack (*Electromet Review*, Vol. XVII, No. 5).—In Bridgeport, Connecticut, there's a smokestack that does its job lying down instead of standing up. It's 100 ft. long; and, besides its horizontal position, it has another interesting feature. It is made of stainless steel.

There are good reasons for both of these construction features. The problem arose when the Southern New England Telephone Company added two-and-a-half stories to its building and then found that employees had smoke in their eyes. A smokestack, rising from the roof of the Post Publishing Company building next door, was spewing out smoke less than 20 ft. from the new windows.

The problem was solved by extending the stack horizontally for 100 ft. to get its discharge end as far as possible from the windows. Stainless steel was used to build the extension because its durability and resistance to corrosion assure long service with little maintenance. Only one end of the stack is fixed, and the supports are placed off-center so that they become vertical as the stack expands thermally. The entire structure was build and erected by The McCathron Boiler Works Company, Bridgeport.

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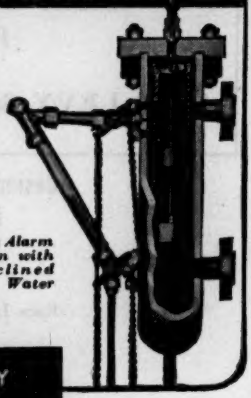

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that are needed, and making them
better . . . whether for peace
or war.

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